

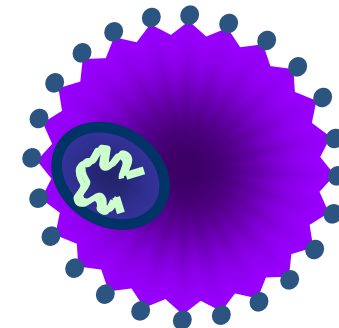
In-host Models

Jane Heffernan

MI2 Lab

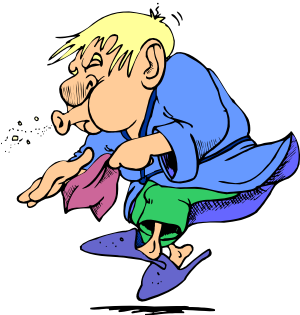
Centre for Disease Modelling

York University



Immune System

- Works around the clock in thousands of different ways, largely unnoticed.
- When it fails, we notice
 - Get sick i.e. bacteria, viruses
- Also notice when it is at work
 - Splinter – inflammation and pain, puss
 - Mosquito bite - get a red, itchy bump
 - Infectious disease – fever, runny nose, cough - the immune system at work!!



Components of Immune system

- Skin - primary boundary between germs and your body
 - Epidermis contains special cells that are an important early-warning component in the immune system.
 - Skin also secretes antibacterial substances- most bacteria and spores that land on the skin die quickly.
- Nose, mouth and eyes - obvious entry points for germs
 - Tears and mucus contain an enzyme that breaks down the cell wall of many bacteria.
 - Saliva is also anti-bacterial.
 - Nasal passage and lungs are coated in mucus - many germs not killed immediately are trapped in the mucus and soon swallowed.
 - Mast cells also line the nasal passages, throat, lungs and skin
- Once inside the body, a germ deals with the immune system at a different level. The major components of the immune system are:
 - Thymus
 - Spleen
 - Lymph system
 - Bone marrow
 - White blood cells
 - Antibodies
 - Complement system
 - Hormones

Immune System

- Each day you
 - Inhale or eat thousands of germs (bacteria and viruses).
 - Your immune system deals with all of them without a problem – skin, mucous, saliva.
 - Occasionally a germ gets past the immune system and you get sick
 - Fever, runny nose, vomiting.
 - If you get better, your immune system was working.
- There are also human ailments that are caused by the immune system working in unexpected or incorrect ways that cause problems.
 - Examples:
 - Allergies - immune system overreacting to certain stimuli that other people don't react to at all.
 - Diabetes - caused by the immune system inappropriately attacking cells in the pancreas and destroying them.
 - Rheumatoid arthritis - caused by the immune system acting inappropriately in the joints
- Finally, immune system may sometimes prevent us from doing things that would be otherwise beneficial
 - i.e. Organ transplants - immune system often rejects the transplanted organ.

Bacteria and Viruses

- Your body
 - Multi-cellular organism made up of perhaps 100 trillion cells.
 - The cells in your body are fairly complicated machines - have a nucleus, energy production equipment, etc.
- Bacteria
 - Single-celled organisms that are much simpler (no nucleus)
 - About 1/100th the size of a human cell (1 micrometer long)
 - Can eat and reproduce
 - One bacterium divides into two separate bacteria perhaps once every 20-30 min
 - At that rate, one bacteria can become millions in just a few hours.
- Virus
 - Not really alive – virus particle is a fragment of DNA/RNA in a protective coat
 - Virus comes in contact with a cell, attaches itself to the cell wall and injects its DNA (and perhaps a few enzymes) into the cell
 - The DNA uses the machinery inside the living cell to reproduce new virus particles
 - Eventually the hijacked cell dies and bursts, freeing the new virus particles; or the viral particles may bud off of the cell so it remains alive

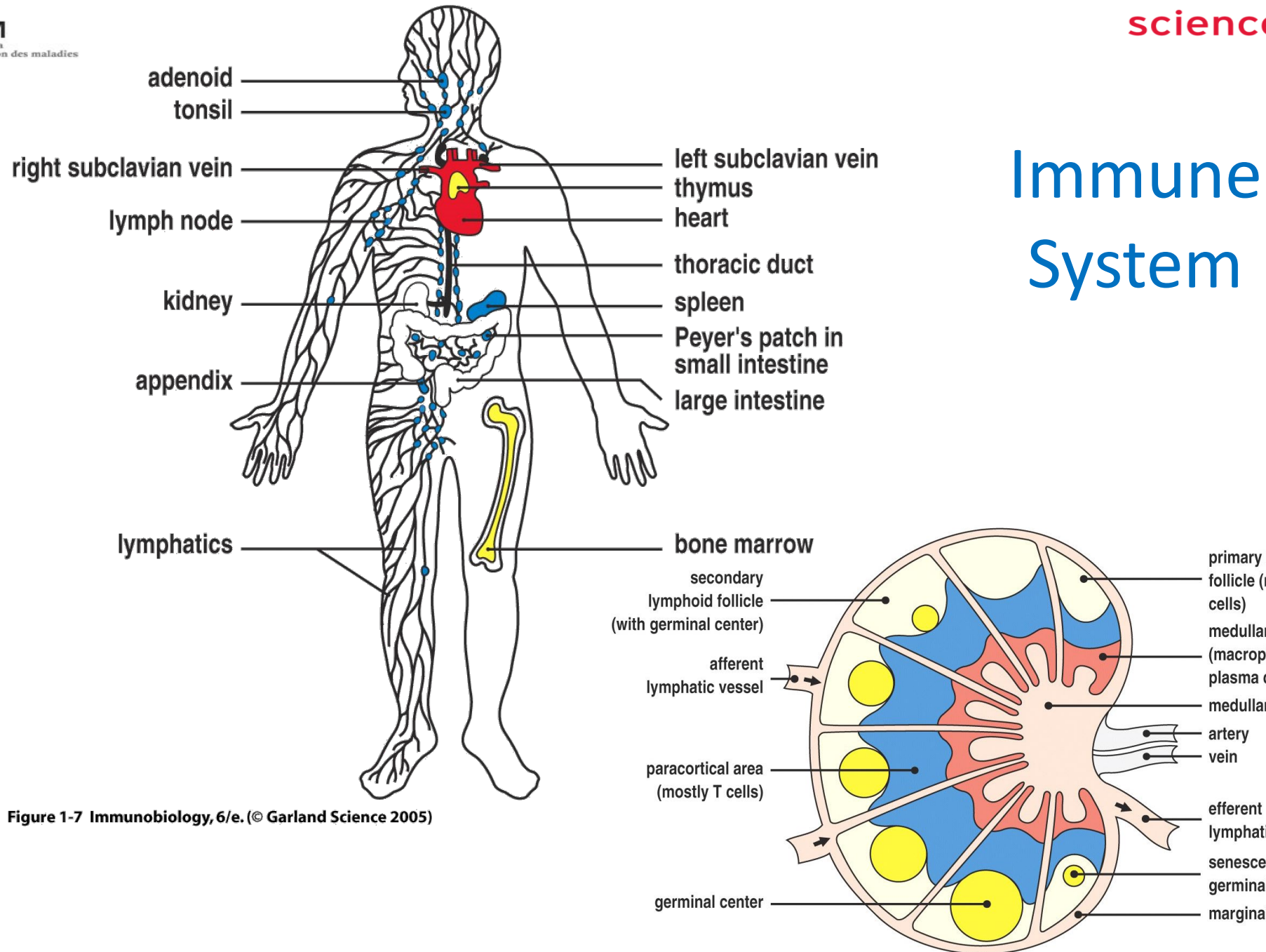


Figure 1-7 Immunobiology, 6/e. (© Garland Science 2005)

Immune System

- Like, the circulatory system, the lymphatic system is a network of vessels.
- Unlike the circulatory system, the lymphatic system has no "pump" located within the lymph vessels
 - Relies on bodily movement as well as nearby pulses from other vessels and organs in the body to facilitate the flow of the lymph fluid.
- I.e. if you do not move, the lymph does not move well.
- Very sedentary individuals – more prone to sickness

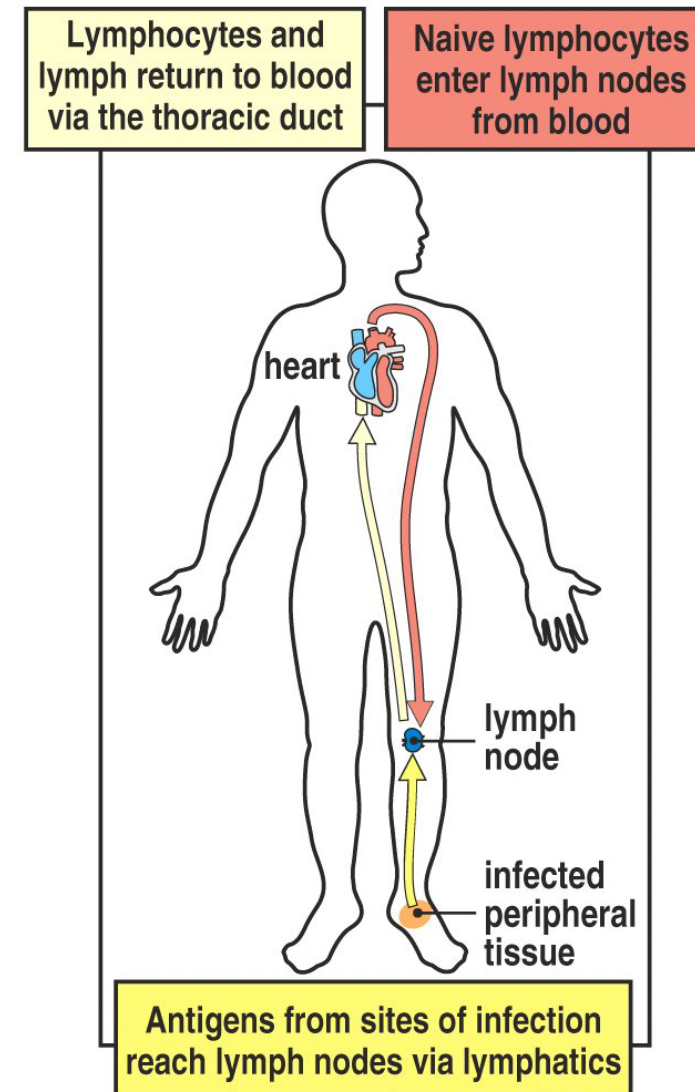


Figure 1-11 Immunobiology, 6/e. (© Garland Science 2005)

Figure 1-3 part 2 of 4

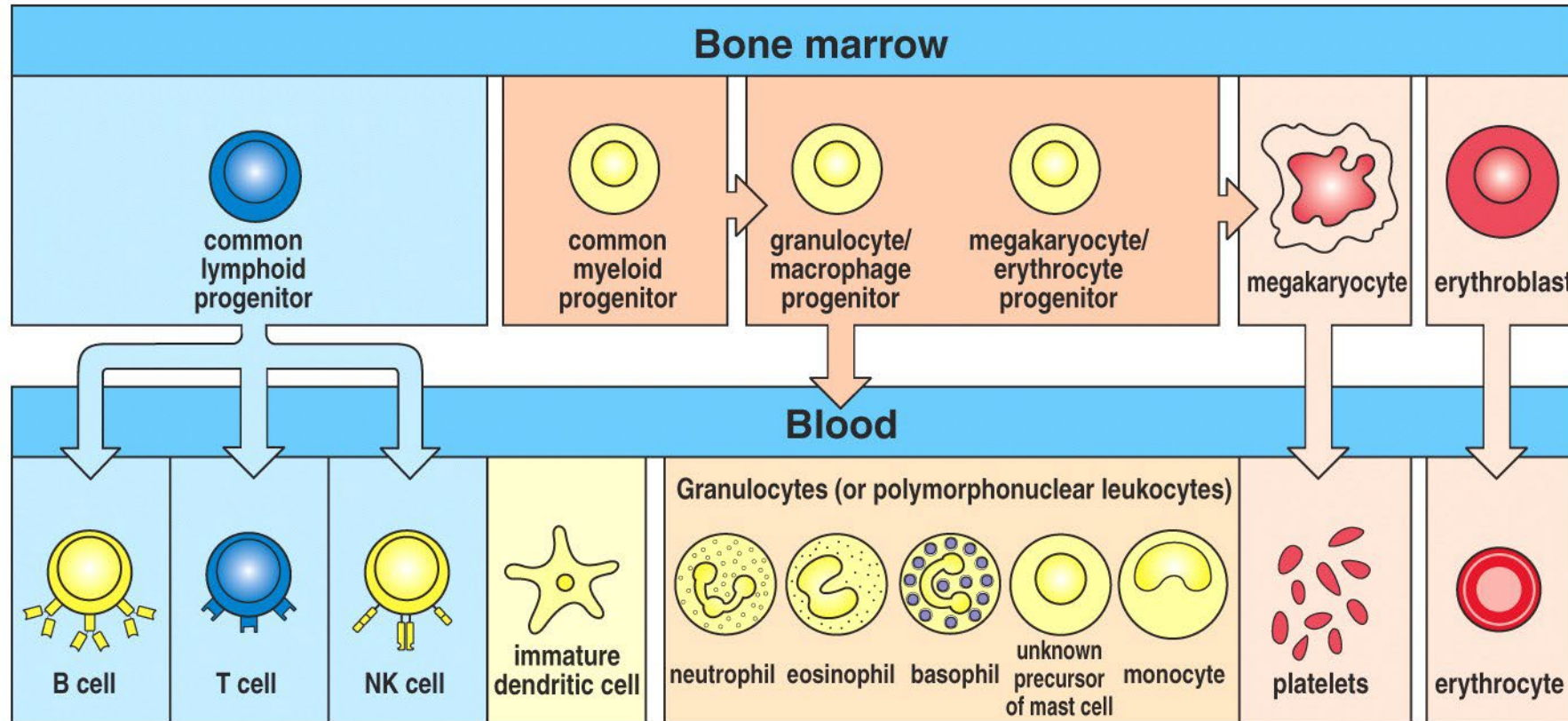


Figure 1-3 part 2 of 4 Immunobiology, 6/e. (© Garland Science 2005)

Figure 1-3 part 3 of 4

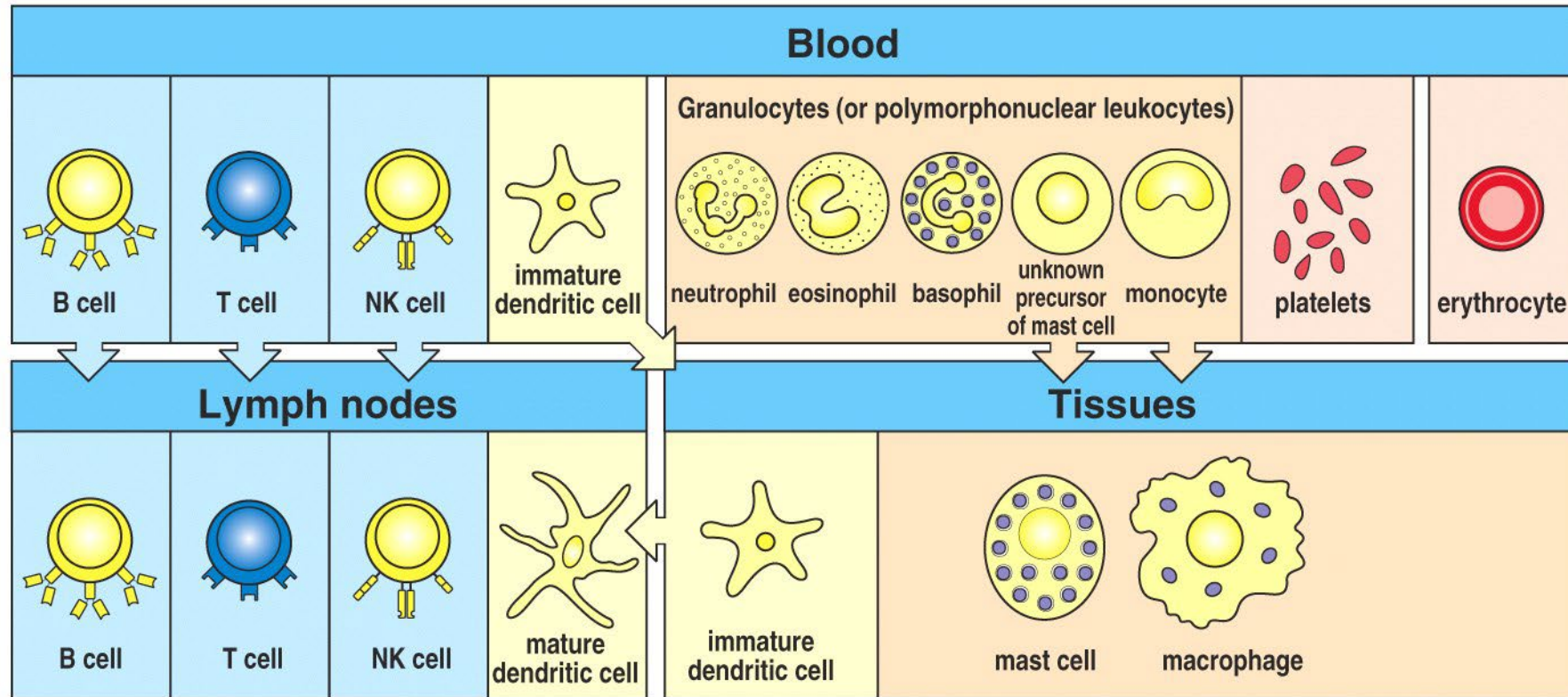


Figure 1-3 part 3 of 4 Immunobiology, 6/e. (© Garland Science 2005)

Figure 1-3 part 4 of 4

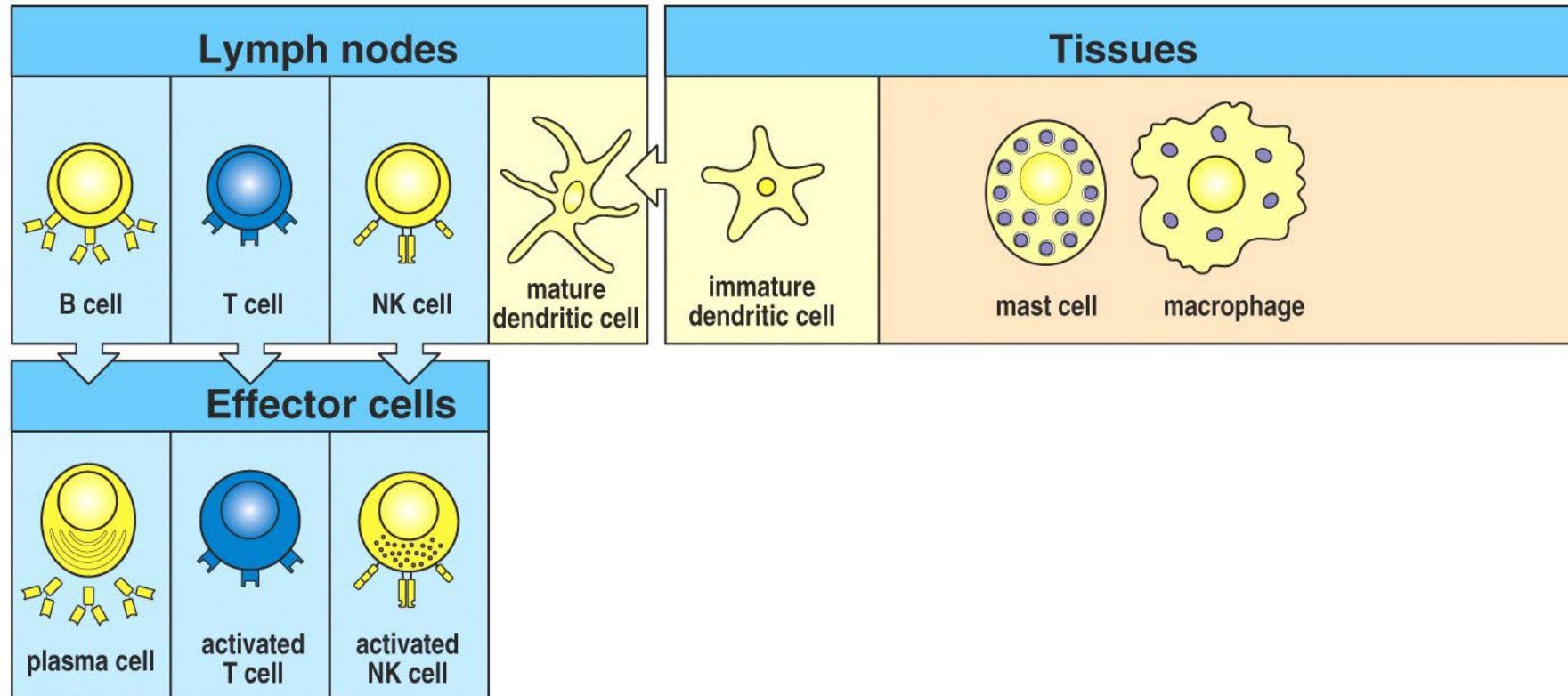
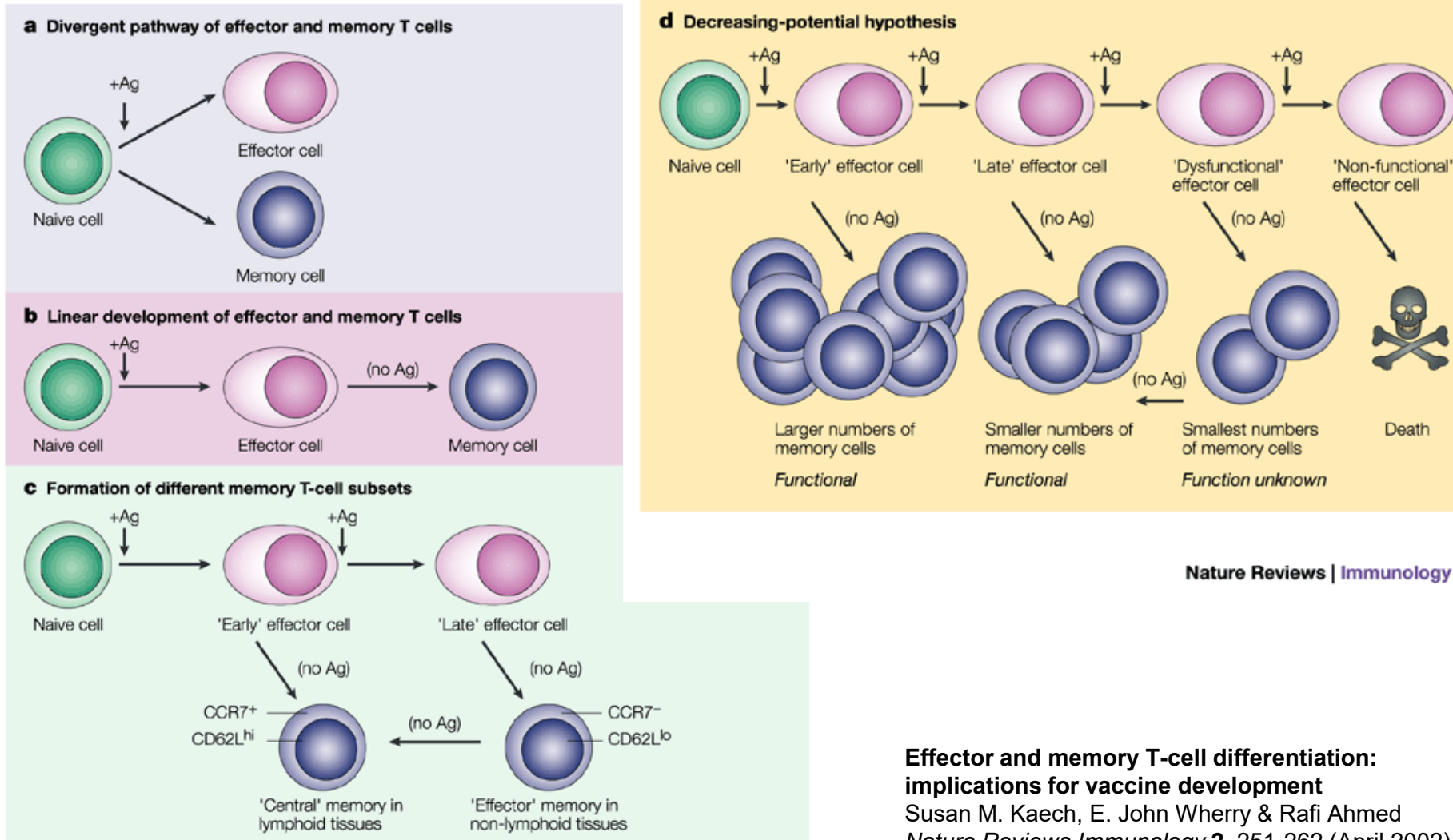


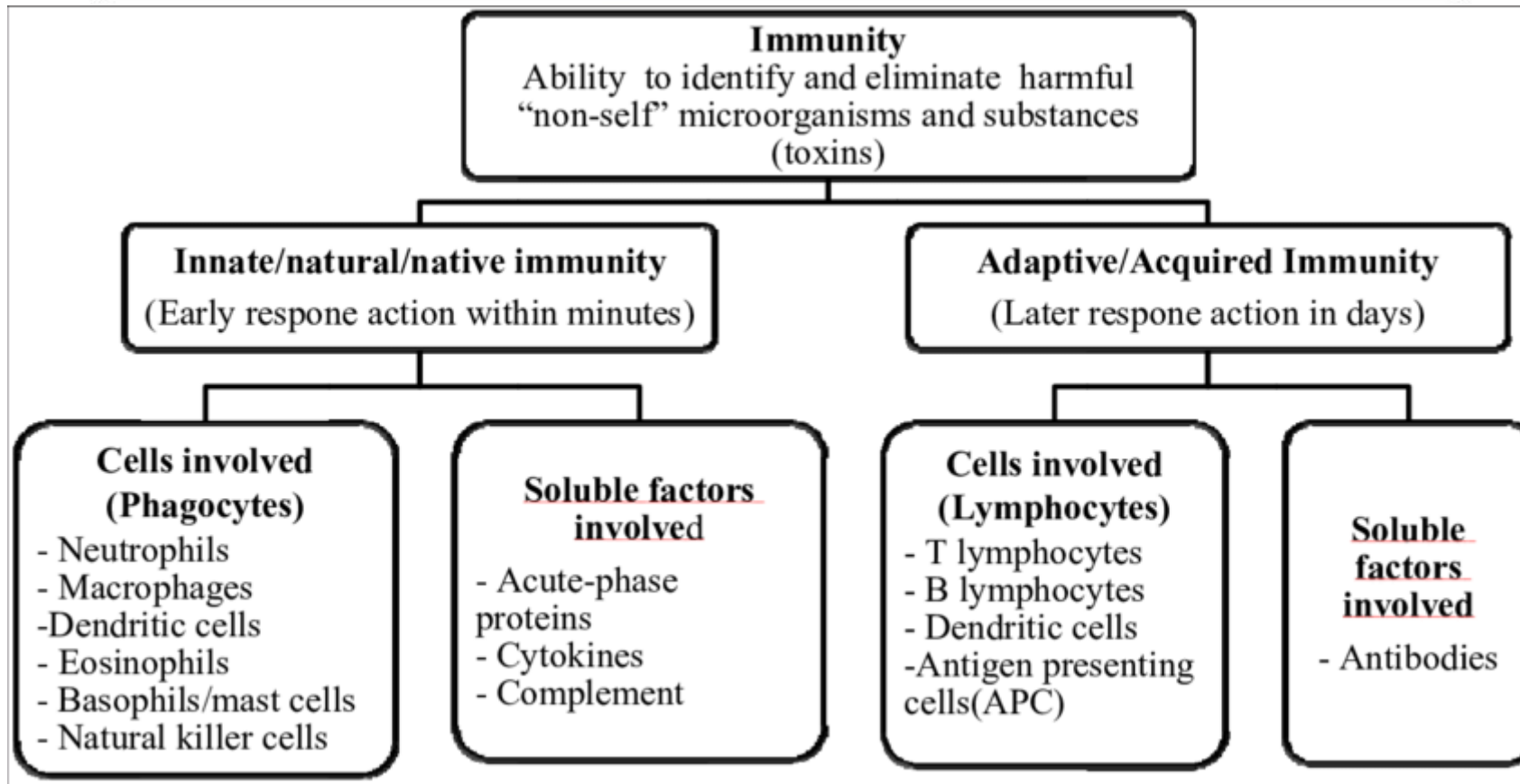
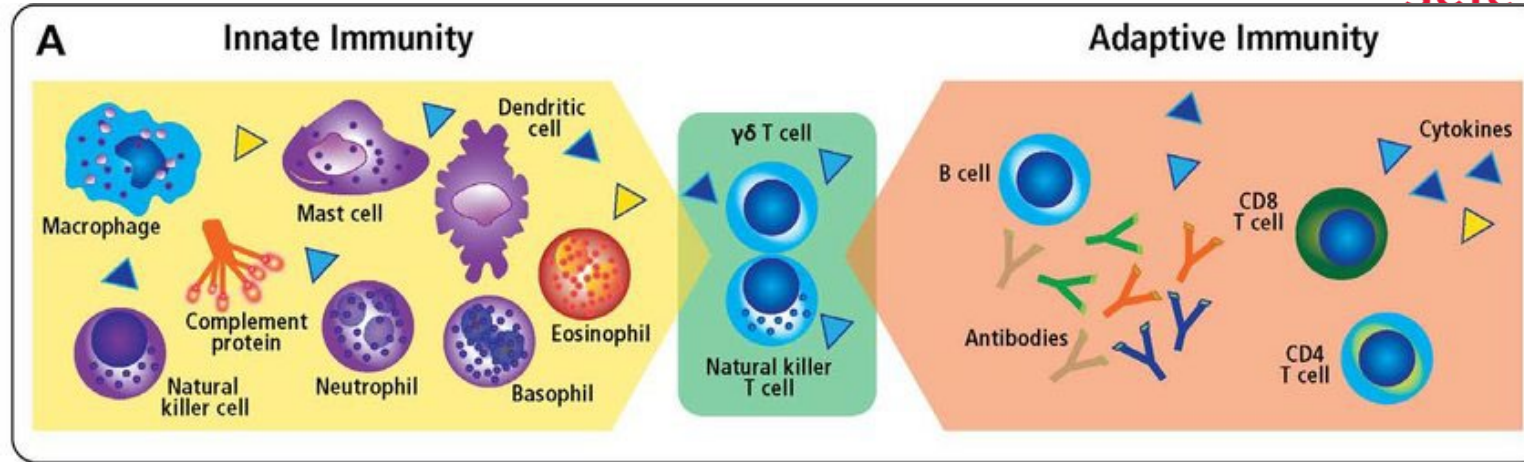
Figure 1-3 part 4 of 4 Immunobiology, 6/e. (© Garland Science 2005)

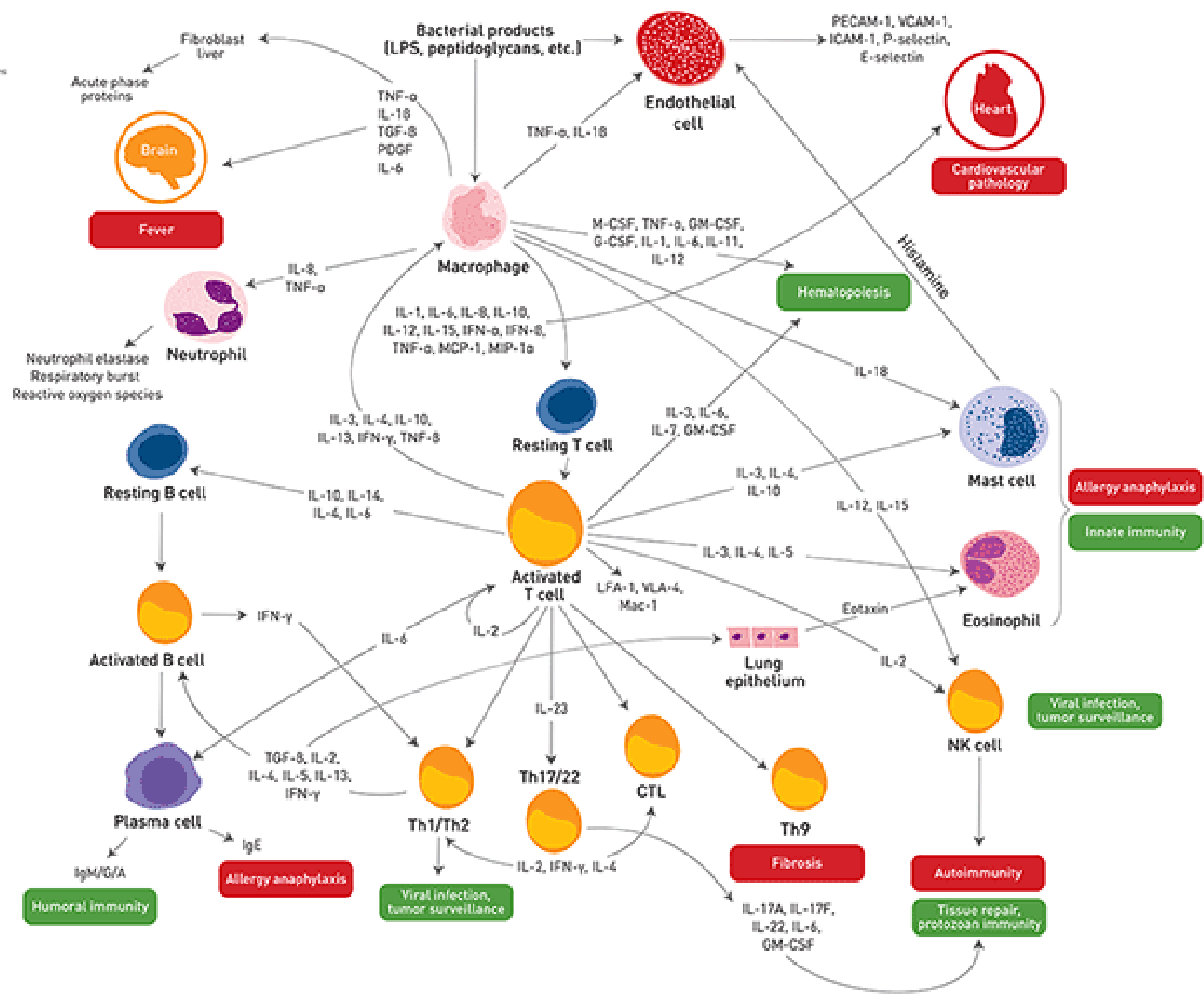
Memory Cells



Nature Reviews | Immunology

Effector and memory T-cell differentiation: implications for vaccine development
 Susan M. Kaech, E. John Wherry & Rafi Ahmed
Nature Reviews Immunology 2, 251-262 (April 2002)





Immune System

- Innate Immunity
- Adaptive Immunity
 - Humoral immunity
 - B-cells and antibodies
 - Cell mediated immunity
 - T-cells
 - Helper and effector
- Lymphocytes
 - T – produced by thymus
 - CD4
 - Helper T-cells
 - Activated by antigen presenting cells
 - Activate the rest of the immune response
 - CD8
 - Killer/effector T-cells
 - B - produced in bone marrow
 - Mature in spleen
 - Plasma cells, antibodies
 - Naive, activated memory

Development of memory

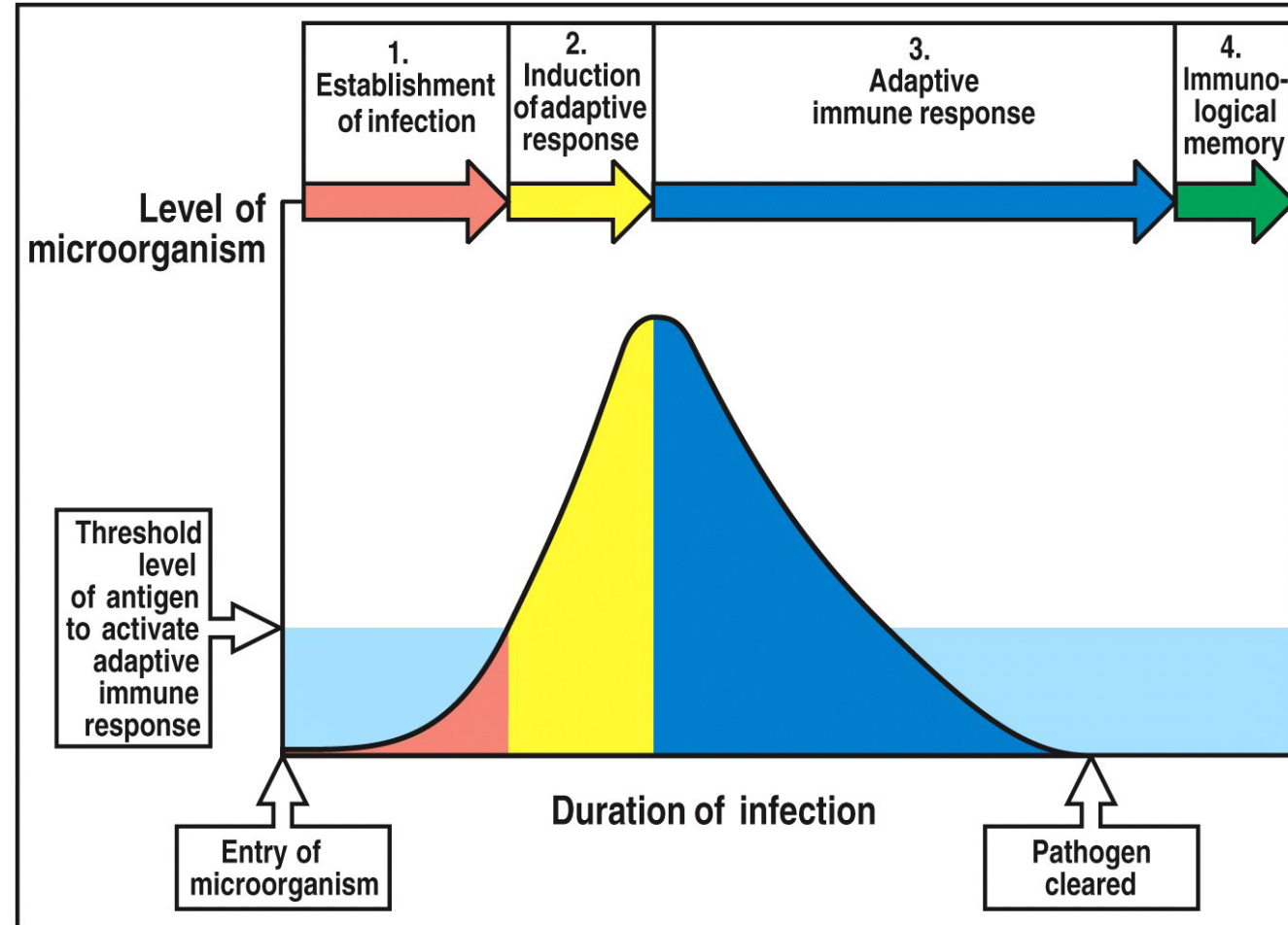


Figure 10-1 Immunobiology, 6/e. (© Garland Science 2005)

- The course of a typical acute infection

Level of immunity

Level of p:
in plasma

Pathogen enters body

Pathogen enters plasma

Infectiousness begins

Symptoms appear

Infectiousness: little to none

Pathogen: clearance or persistence

Establishment of infection

Initiate adaptive immune response

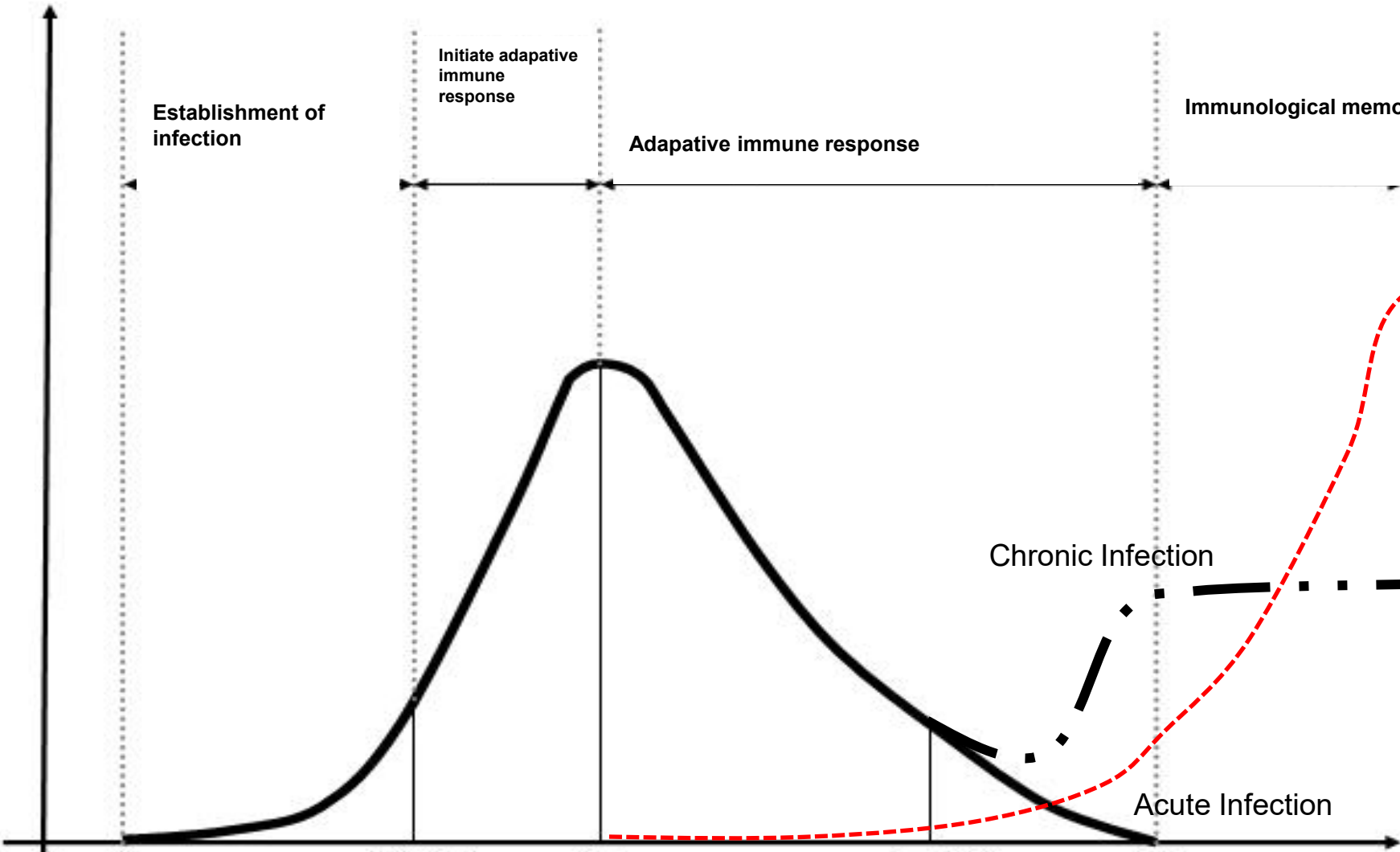
Adaptive immune response

Immunological memory

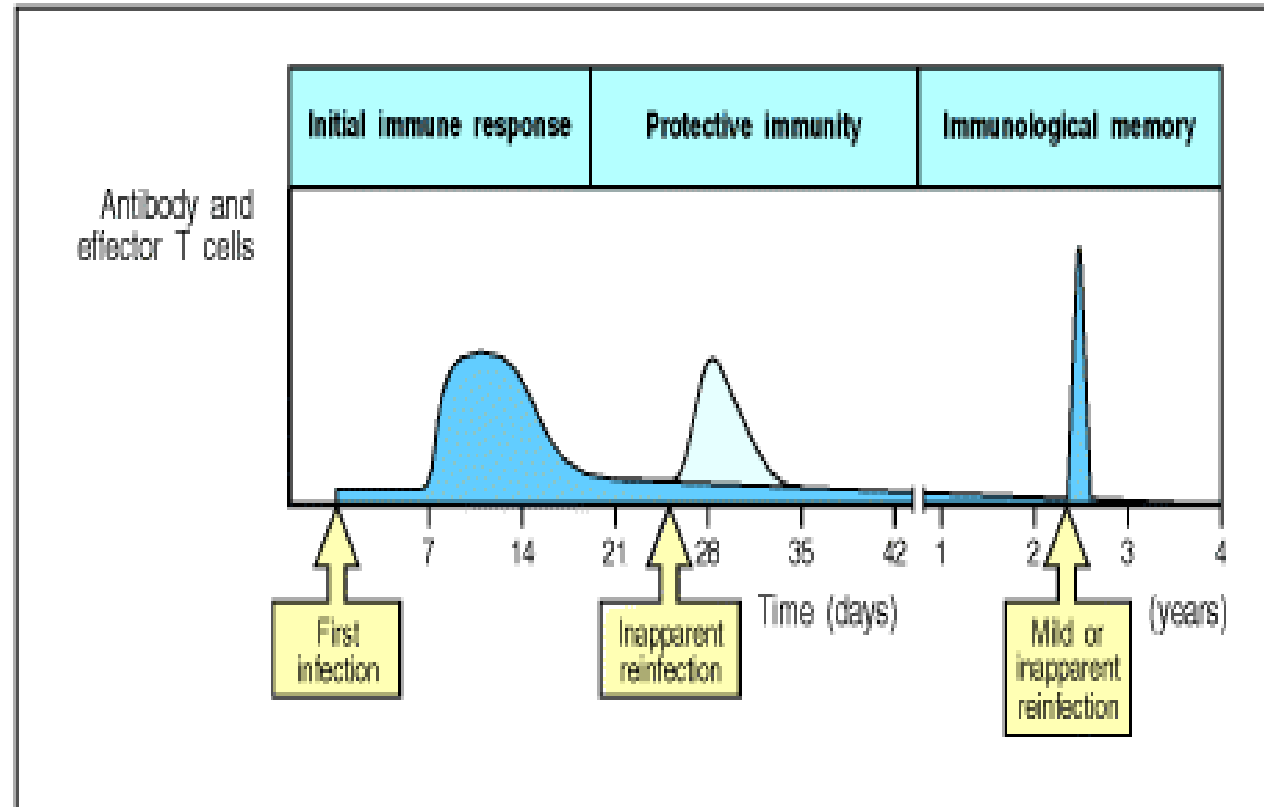
Immune memory

Chronic Infection

Acute Infection



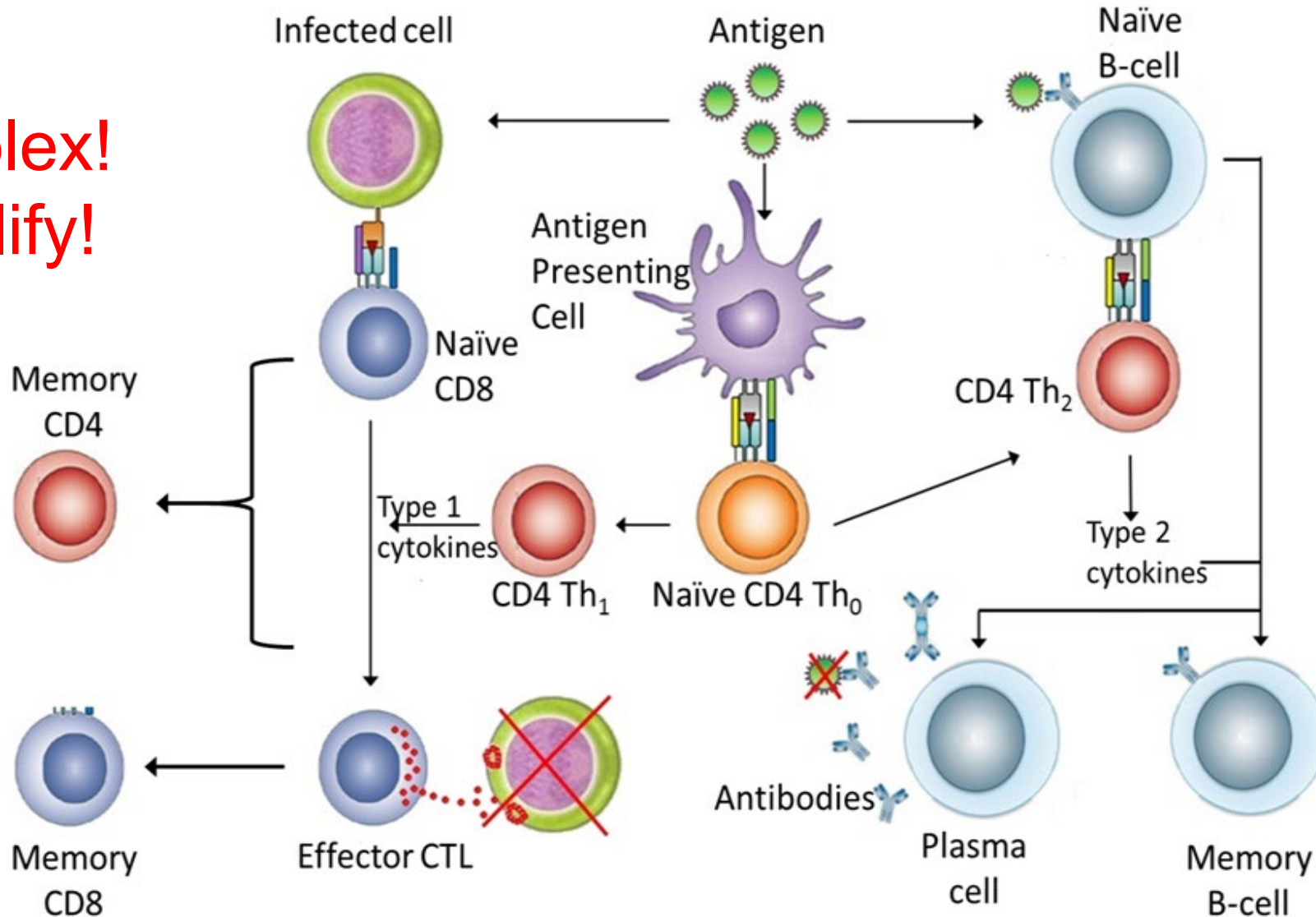
Immunological Memory



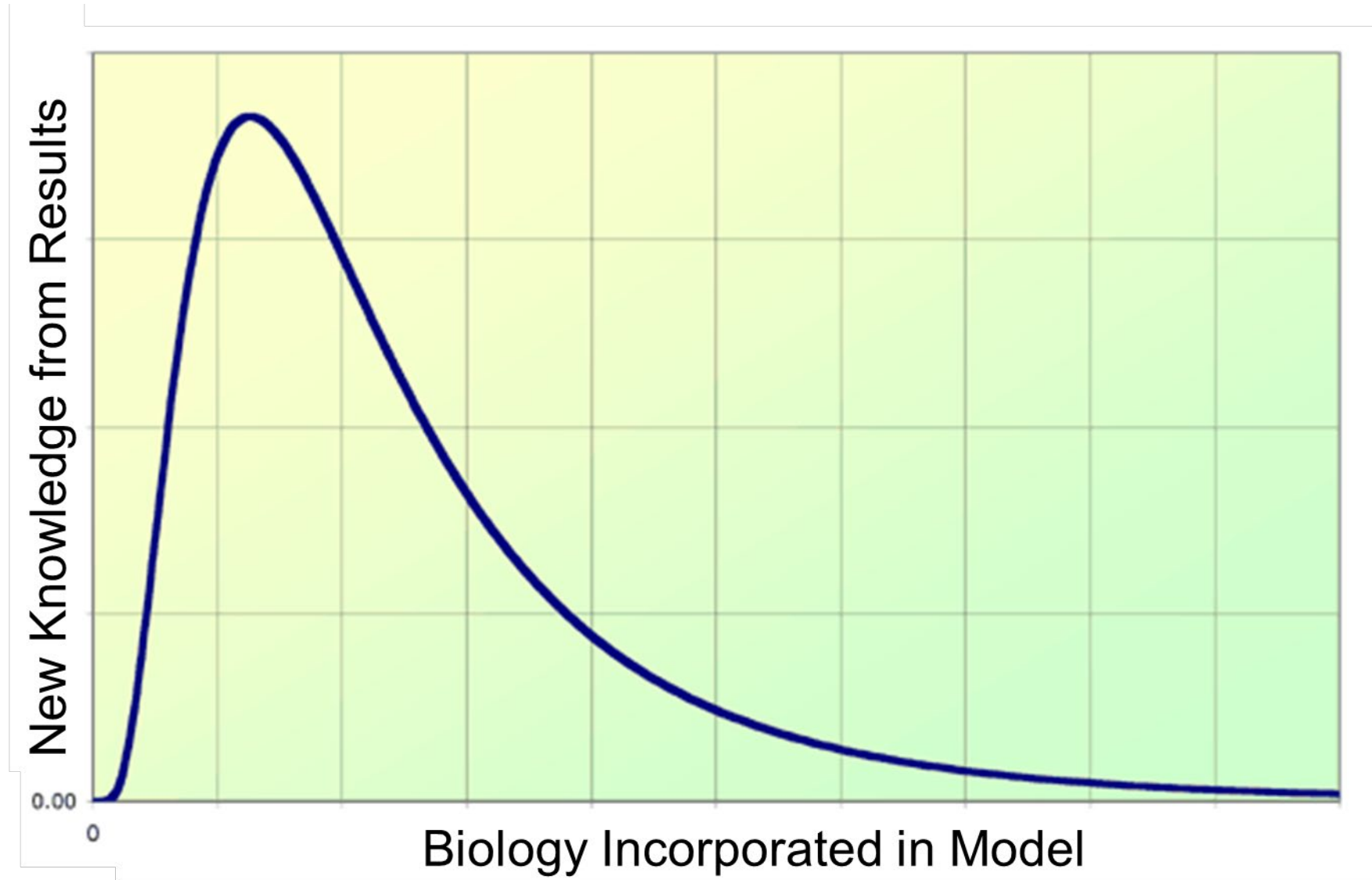
- Protective immunity consists of preformed immune reactants and immunological memory.

Understand immune system/in-host dynamics

Quite complex!
 Simplify!



Adding in uncertainty



Memory Cells

- Long lived
- This can be a good thing (COVID-19, influenza, vaccination)
- This can be a bad thing (HIV...)

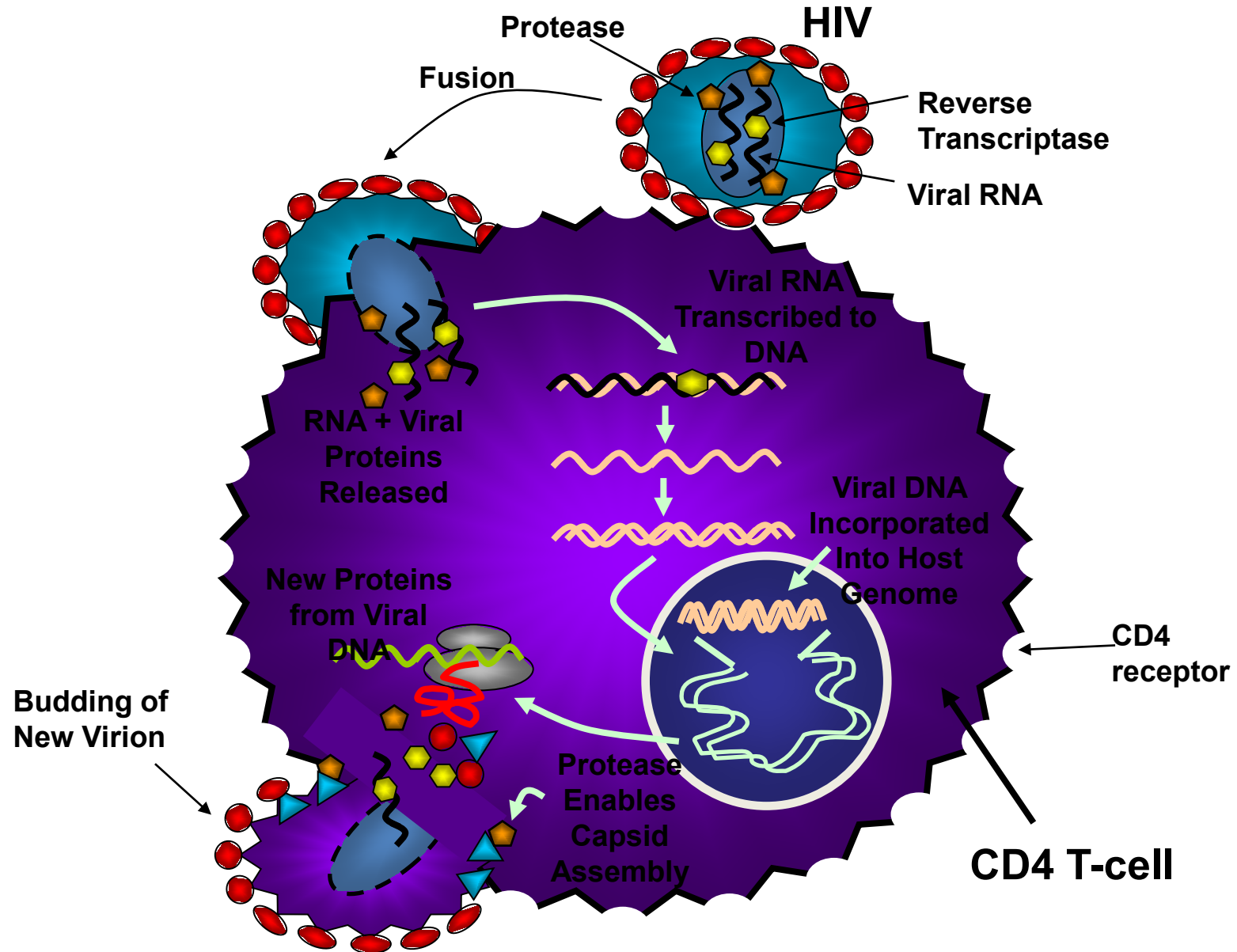
Virus Fitness

- Reproduction number
- Can define fitness for different virus variants
- We will also touch on this today

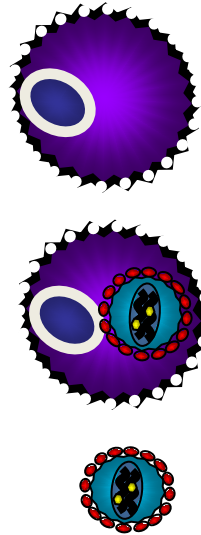
Basic Model

- Applicable to all studies of pathogen dynamics in-host
- Many extensions
- Motivated by HIV

Basic Model



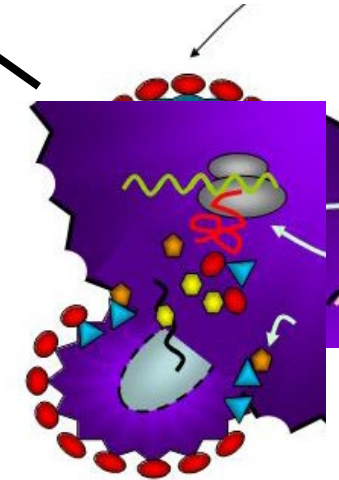
Basic Model



$$\frac{dx}{dt} = \lambda - d_x x - \beta x v$$

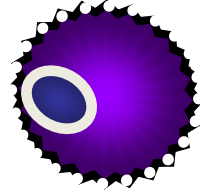
$$\frac{dy}{dt} = \beta x v - d_y y$$

$$\frac{dv}{dt} = k y - d_v v$$

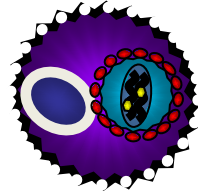


- x - uninfected cells
- y - infected cells
- v - free virus
- λ, k - production rate
- β - efficacy of infection
- d_x, d_y, d_v - death rates/clearance time

Basic Model- with virus loss



$$\frac{dx}{dt} = \lambda - d_x x - \beta x v$$



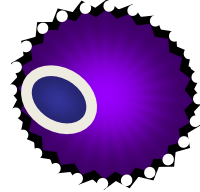
$$\frac{dy}{dt} = \beta x v - d_y y$$



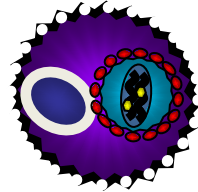
$$\frac{dv}{dt} = k y - d_v v - \beta x v$$

- x - uninfected cells
- y - infected cells
- v - free virus
- λ, k - production rate
- β - efficacy of infection
- d_x, d_y, d_v - death rates/ clearance time

Where is the immune system?



$$\frac{dx}{dt} = \lambda - d_x x - \beta x v$$



$$\frac{dy}{dt} = \beta x v - d_y y$$



$$\frac{dv}{dt} = k y - d_v v - \beta x v$$

- x - uninfected cells
- y - infected cells
- v - free virus
- λ, k - production rate
- β - efficacy of infection
- d_x, d_y, d_v - death rates/
clearance time

Intro Analysis

```

[> restart :
[> with(LinearAlgebra) :
[> eq1 := lambda - dx·x - beta·x·v;
                                eq1 := λ - dx x - β x v
[> eq2 := beta·x·v - dy·y;
                                eq2 := β x v - dy y
[> eq3 := k·q·y - dv·v - beta·x·v;
                                eq3 := k q y - dv v - β x v
[> eq4 := k·(1 - q)·y - dv·w;
                                eq4 := k (1 - q) y - dv w
  
```

> $equil := solve(\{eq1 = 0, eq2 = 0, eq3 = 0, eq4 = 0\}, \{x, y, v, w\});$

$$equil := \left\{ v = 0, w = 0, x = \frac{\lambda}{dx}, y = 0 \right\}, \left\{ v = -\frac{-\lambda \beta k q + \lambda \beta dy + dx dv dy}{dv dy \beta}, w = \right.$$

$$\left. -\frac{k(-\lambda \beta k q + \lambda \beta dy + dx dv dy)(-1 + q)}{\beta dy(-k q + dy) dv}, x = -\frac{dv dy}{\beta(-k q + dy)}, y = \right.$$

$$\left. = \frac{-\lambda \beta k q + \lambda \beta dy + dx dv dy}{\beta dy(-k q + dy)} \right\}$$

> $J := Matrix(4, 4, [[diff(eq1, x), diff(eq1, y), diff(eq1, v), diff(eq1, w)], [diff(eq2, x), diff(eq2, y), diff(eq2, v), diff(eq2, w)], [diff(eq3, x), diff(eq3, y), diff(eq3, v), diff(eq3, w)], [diff(eq4, x), diff(eq4, y), diff(eq4, v), diff(eq4, w)]]);$

$$J := \begin{bmatrix} -dx - \beta v & 0 & -\beta x & 0 \\ \beta v & -dy & \beta x & 0 \\ -\beta v & k q & -dv - \beta x & 0 \\ 0 & k(1 - q) & 0 & -dv \end{bmatrix}$$

$$\begin{aligned}
 &> \text{equil} := \text{solve}(\{eq1 = 0, eq2 = 0, eq3 = 0, eq4 = 0\}, \{x, y, v, w\}); \\
 \text{equil} &:= \left\{ v = 0, w = 0, x = \frac{\lambda}{dx}, y = 0 \right\}, \left\{ v = - \frac{-\lambda \beta k q + \lambda \beta dy + dx dv dy}{dv dy \beta}, w = \right. \\
 &\quad \left. - \frac{k (-\lambda \beta k q + \lambda \beta dy + dx dv dy) (-1 + q)}{\beta dy (-k q + dy) dv}, x = - \frac{dv dy}{\beta (-k q + dy)}, y \right. \\
 &\quad \left. = \frac{-\lambda \beta k q + \lambda \beta dy + dx dv dy}{\beta dy (-k q + dy)} \right\}
 \end{aligned}$$

$$\begin{aligned}
 &> J := \text{Matrix}(4, 4, [[\text{diff}(eq1, x), \text{diff}(eq1, y), \text{diff}(eq1, v), \text{diff}(eq1, w)], [\text{diff}(eq2, x), \\
 &\quad \text{diff}(eq2, y), \text{diff}(eq2, v), \text{diff}(eq2, w)], [\text{diff}(eq3, x), \text{diff}(eq3, y), \text{diff}(eq3, v), \text{diff}(eq3, \\
 &\quad w)], [\text{diff}(eq4, x), \text{diff}(eq4, y), \text{diff}(eq4, v), \text{diff}(eq4, w)]]);
 \end{aligned}$$

$$J := \begin{bmatrix} -dx - \beta v & 0 & -\beta x & 0 \\ \beta v & -dy & \beta x & 0 \\ -\beta v & kq & -dv - \beta x & 0 \\ 0 & k(1 - q) & 0 & -dv \end{bmatrix}$$

> *eigvals* := Eigenvalues(subs({x = x0, y = 0, v = 0, w = 0}, J));

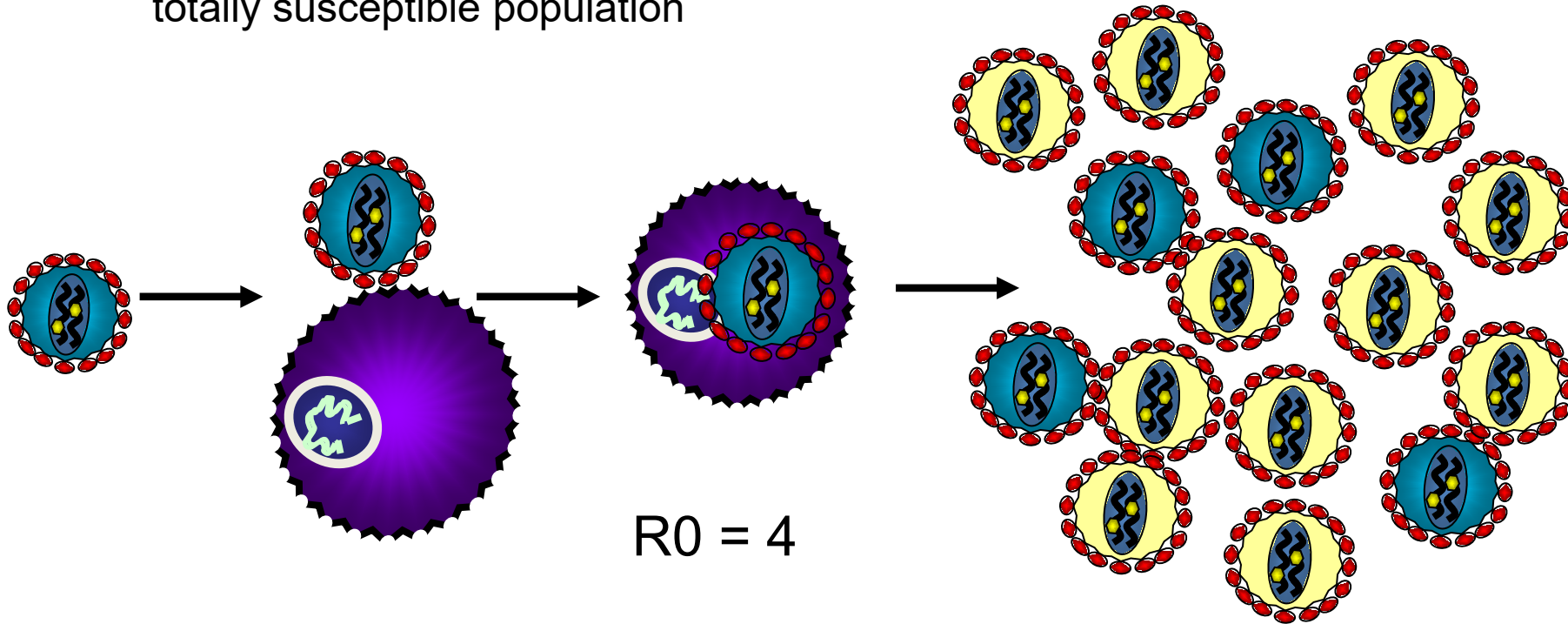
$$\begin{aligned}
 \mathit{eigvals} := & \left[\begin{array}{l} -dx \\ -dv \\ -\frac{1}{2} dv - \frac{1}{2} dy - \frac{1}{2} \beta x_0 \\ + \frac{1}{2} \sqrt{dv^2 - 2 dv dy + 2 dv \beta x_0 + dy^2 - 2 dy \beta x_0 + \beta^2 x_0^2 + 4 k q \beta x_0} \\ -\frac{1}{2} dv - \frac{1}{2} dy - \frac{1}{2} \beta x_0 \\ - \frac{1}{2} \sqrt{dv^2 - 2 dv dy + 2 dv \beta x_0 + dy^2 - 2 dy \beta x_0 + \beta^2 x_0^2 + 4 k q \beta x_0} \end{array} \right]
 \end{aligned}$$

> solve(*eigvals*[3] = 0, k) : R0 := $\frac{k}{\%}$;

$$R0 := \frac{k q \beta x_0}{dy (dv + \beta x_0)}$$

In-host Model

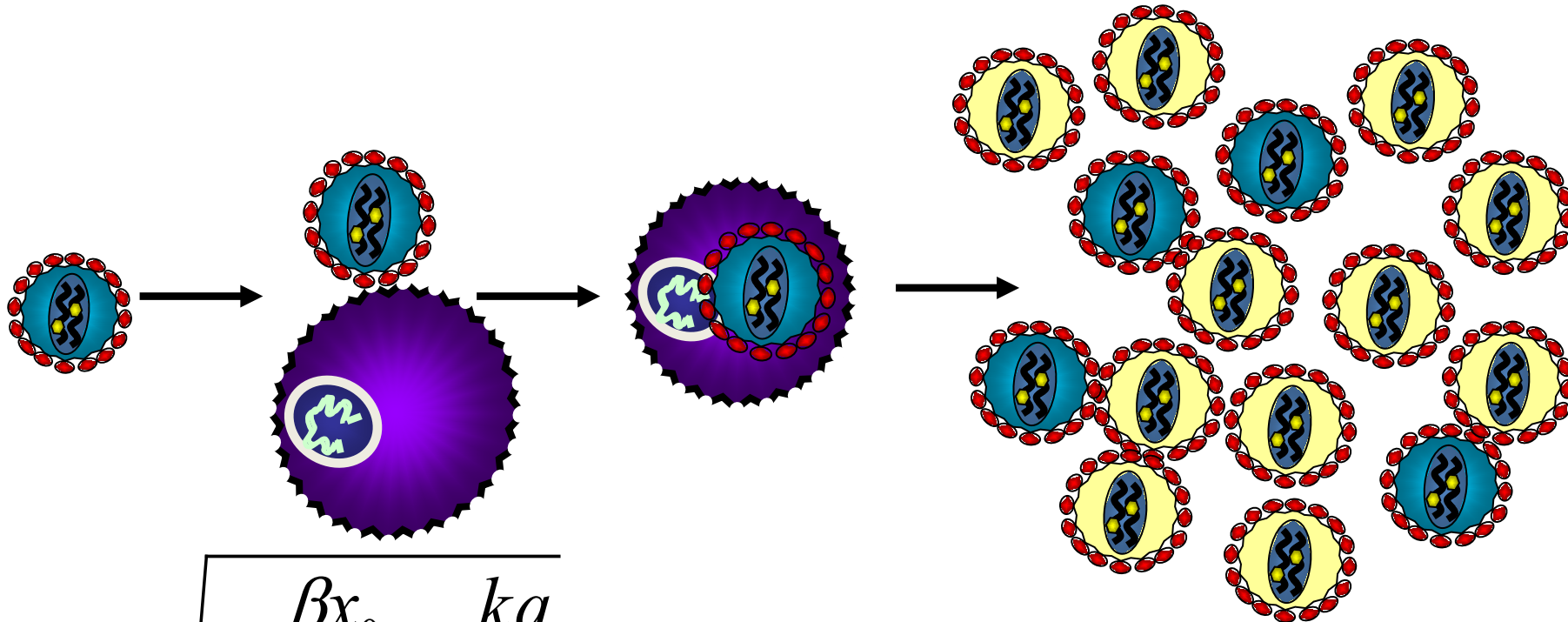
- Basic reproductive ratio
 - The number of secondary infections produced by an initial infective in a totally susceptible population



$$R_0 = \frac{\beta x_0}{u + \beta x_0} \frac{kq}{d_y}$$

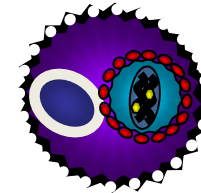
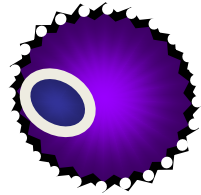
In-host Model

- Basic reproductive ratio
 - The number of secondary infections produced by an initial infective in a totally susceptible population



$$R_0 = \sqrt{\frac{\beta x_0}{d_v + \beta x_0} \frac{kq}{d_y}}$$

Basic Model- with drugs



$$\frac{dx}{dt} = \lambda - d_x x - \beta(1 - \epsilon_{rt})xv$$

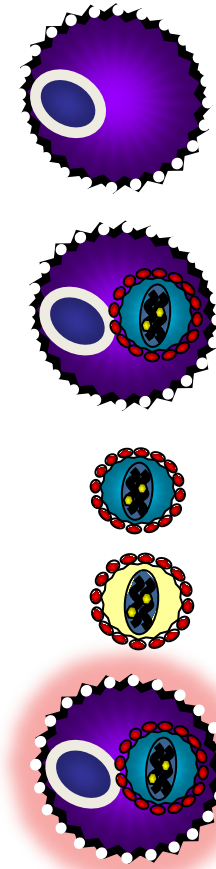
$$\frac{dy}{dt} = \beta(1 - \epsilon_{rt})xv - d_y y$$

$$\frac{dv}{dt} = kq(1 - \epsilon_p)y - d_v v - \beta xv$$

$$\frac{dw}{dt} = k(1 - q(1 - \epsilon_p))y - d_v w$$

$$R_c = \frac{\beta(1 - \epsilon_{rt})x_0}{\beta x_0 + d_v} \frac{kq(1 - \epsilon_p)}{d_y}$$

Latently Infected Cells



$$x' = \lambda - d_x x - \beta x v$$

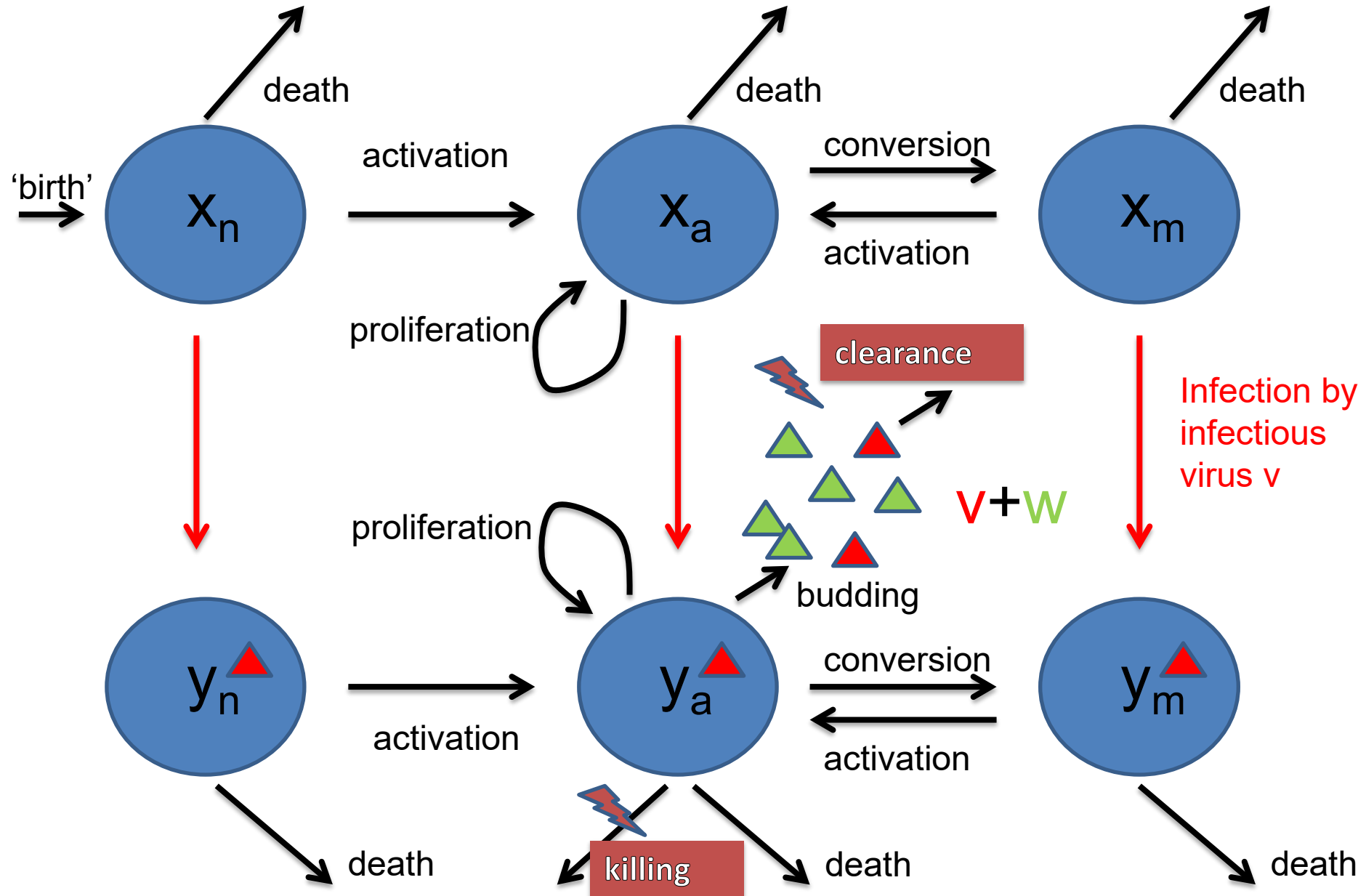
$$y' = (1 - f)\beta x v - d_y y + \psi l$$

$$v' = k q y - d_v v - \beta x v$$

$$w' = k(1 - q)y - d_v w$$

$$l' = f\beta x v - d_l l - \psi l$$

CD4 T-cell immunity & Latently Infected Cells

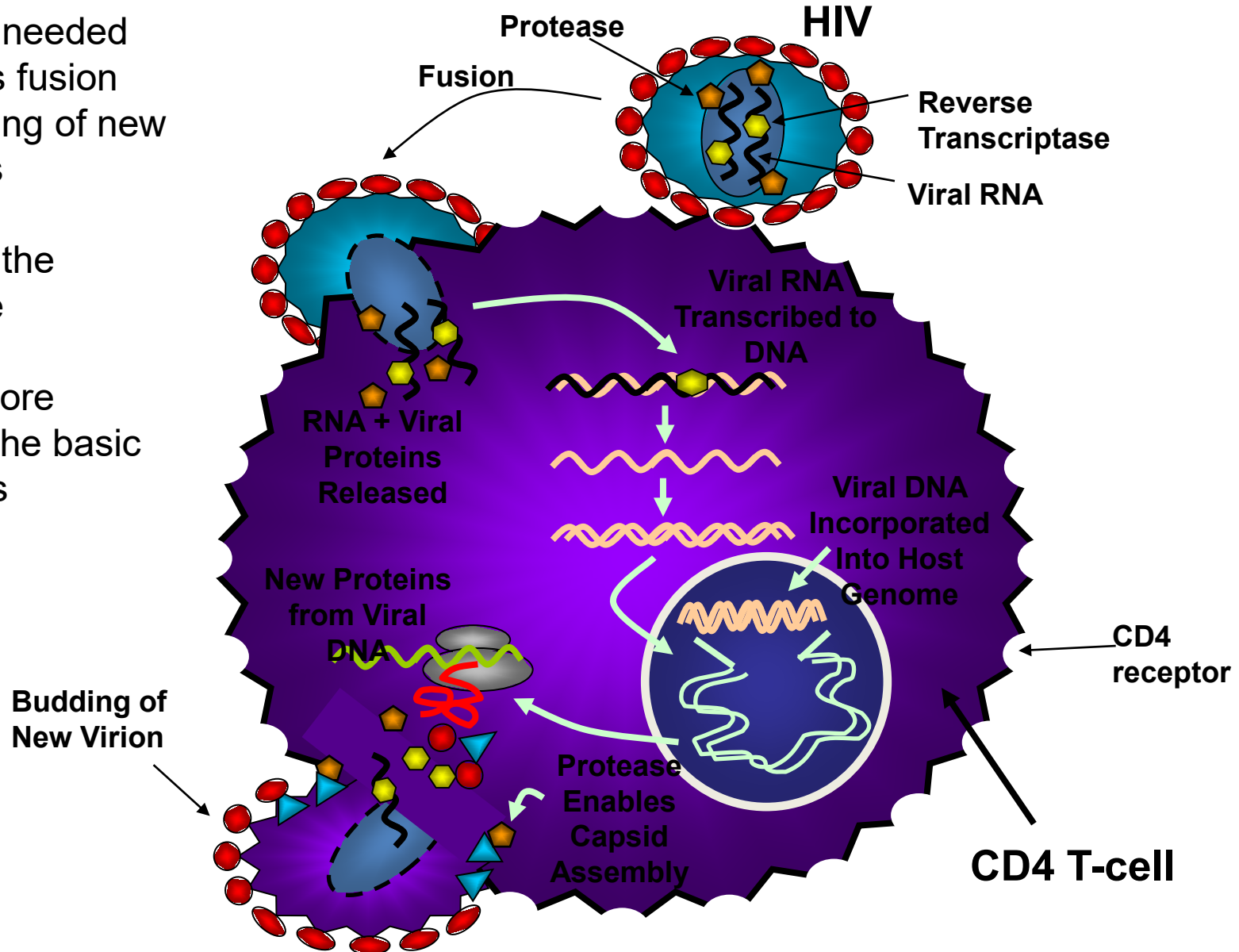


Eclipse Stage

Some time is needed between virus fusion and the budding of new virus particles

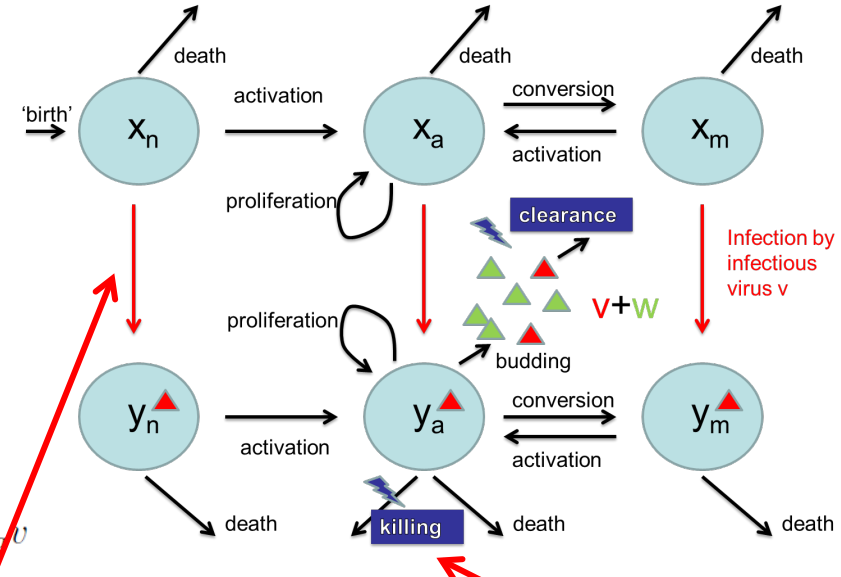
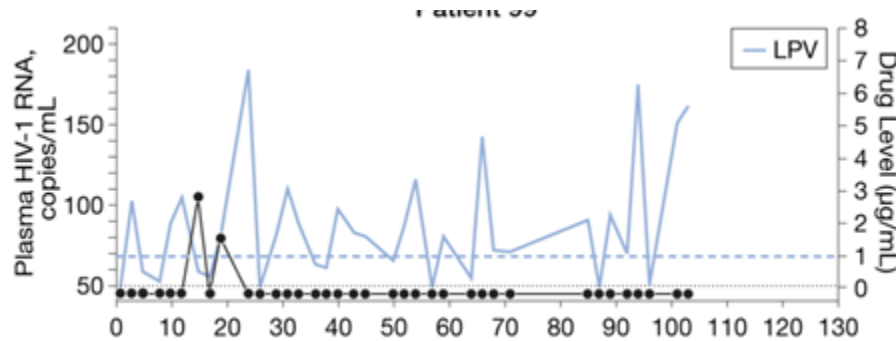
This is called the Eclipse Stage

Add one or more equations to the basic model of virus dynamics



Viral Blips

- ODEs, include long lived memory cells



$$\frac{dx_n}{dt} = \lambda - d_{x_n}x_n - \beta x_n v - \gamma x_n(v + \delta)$$

$$\frac{dx_a}{dt} = \gamma x_n(v + \delta) + cx_m v + \rho x_a \frac{v}{v+C} - d_{x_a}x_a - px_a - \beta x_a v$$

$$\frac{dx_m}{dt} = px_a - cx_m v - d_{x_m}x_m - \beta x_m v$$

$$\frac{dy_a}{dt} = \gamma y_n v + \beta x_a v + \rho y_a \frac{v}{v+C} + cy_m v - d_{y_a}y_a - py_a - \xi y_a f(v, x_a, y_a)$$

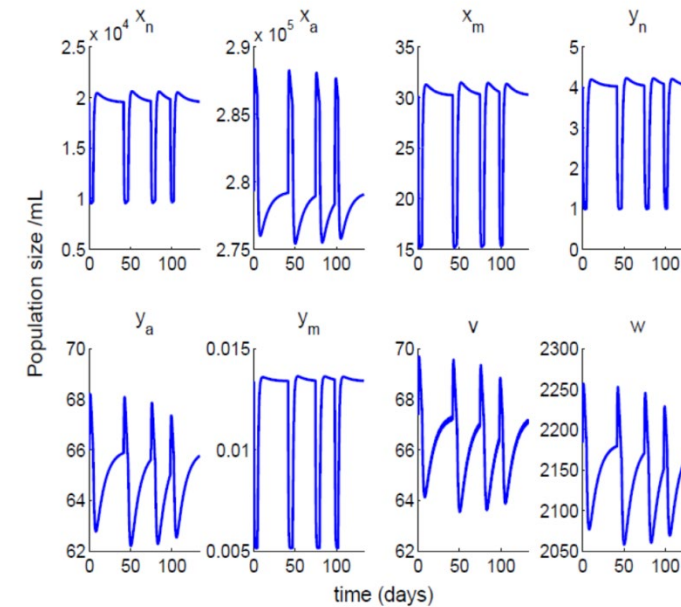
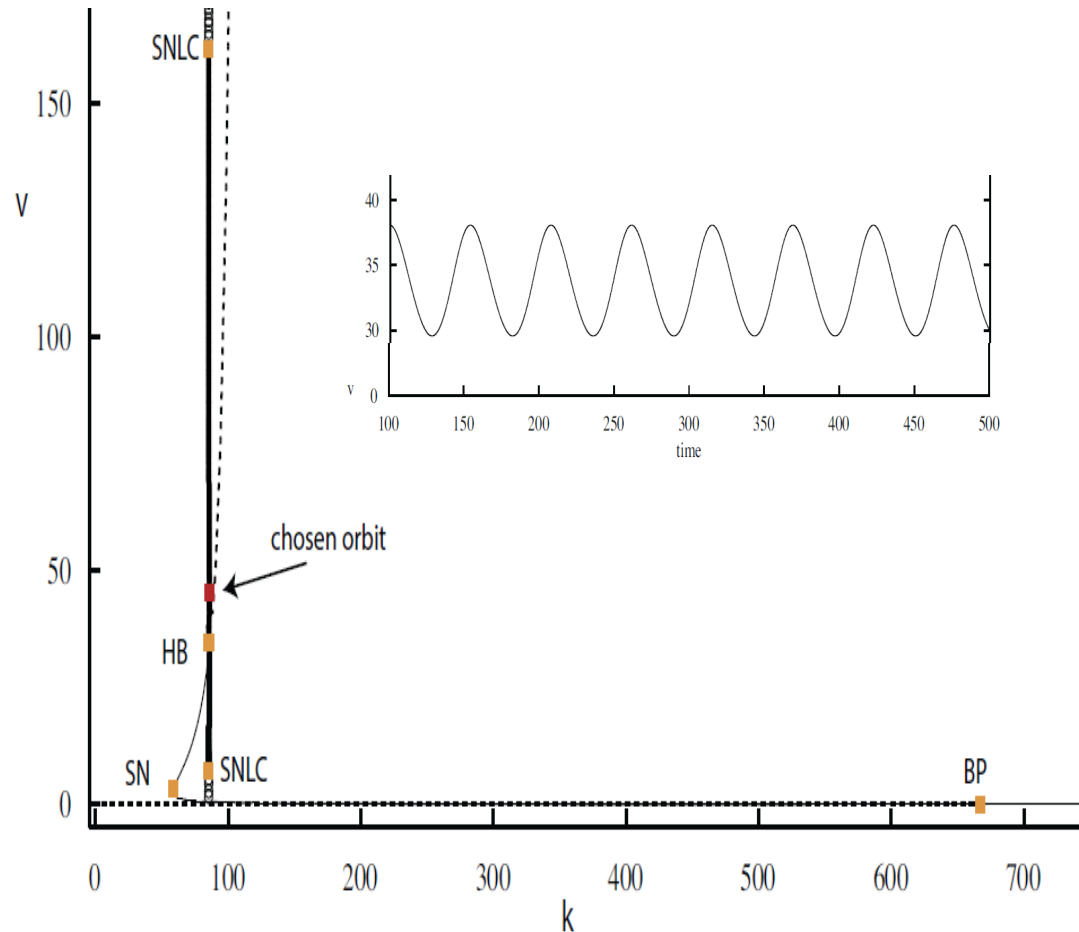
$$\frac{dy_m}{dt} = py_a + \beta x_m v - cy_m v - d_{y_m}y_m$$

$$\frac{dv}{dt} = kqy_a - uv - s\beta v(x_n + x_a + x_m)$$

Frascoli F, Wang Y, et al. (2014). CAMQ, in press.

Viral Blips

- ODEs, include long lived memory cells



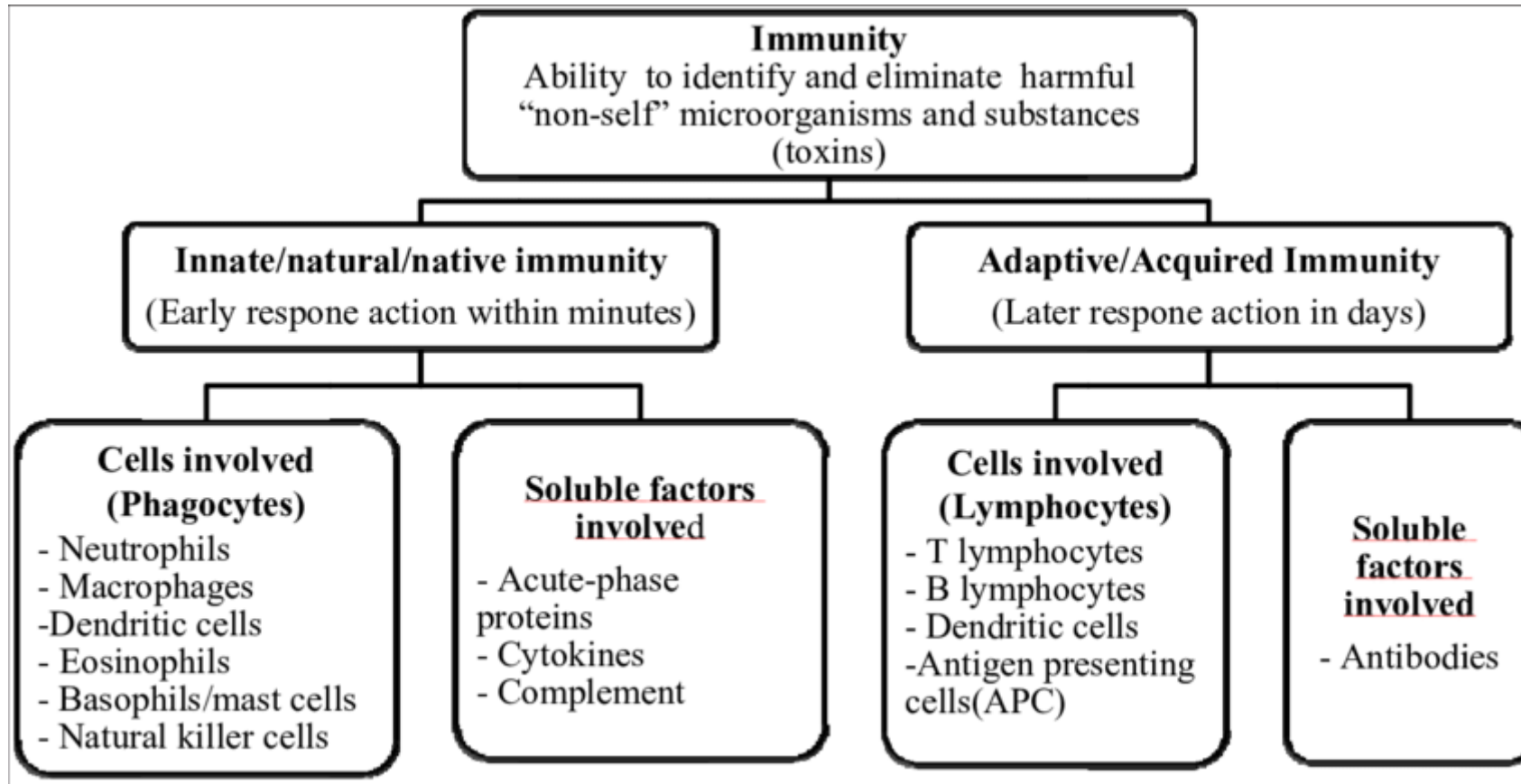
Tat 'on-off' switch

On-going MI2 work

Bifurcations: Backward (1 or 2), Hopf (1 or 2), depend on s and $f(x,y,v)$ Tat 'on-off' switch

Frascoli F, Wang Y, et al. (2014). *CAMQ*, in press.

More to consider...



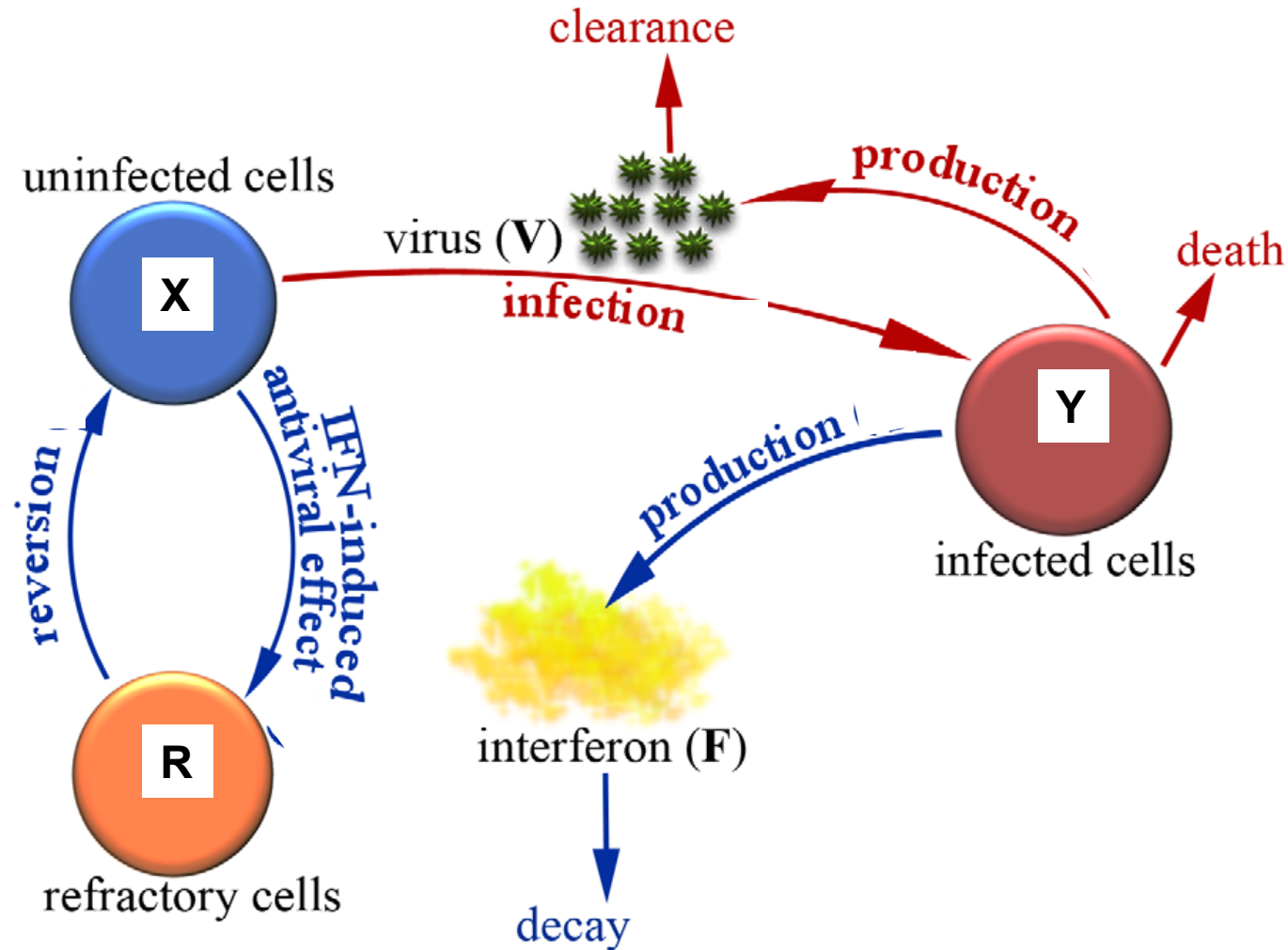
with killer cells and antibodies

$$\begin{aligned}x' &= \lambda - d_x x - \beta x v \\y' &= \beta x v - d_y y - \xi y z \\v' &= k q y - d_v v - \beta x v - \phi a v \\z' &= F(x, y, v, z, a) - d_z z \\a' &= G(x, y, v, z, a) - d_a a\end{aligned}$$

$$R_0 = \frac{\beta x_0}{\beta x_0 + d_v + a_0} \frac{k q}{d_y + z_0}$$

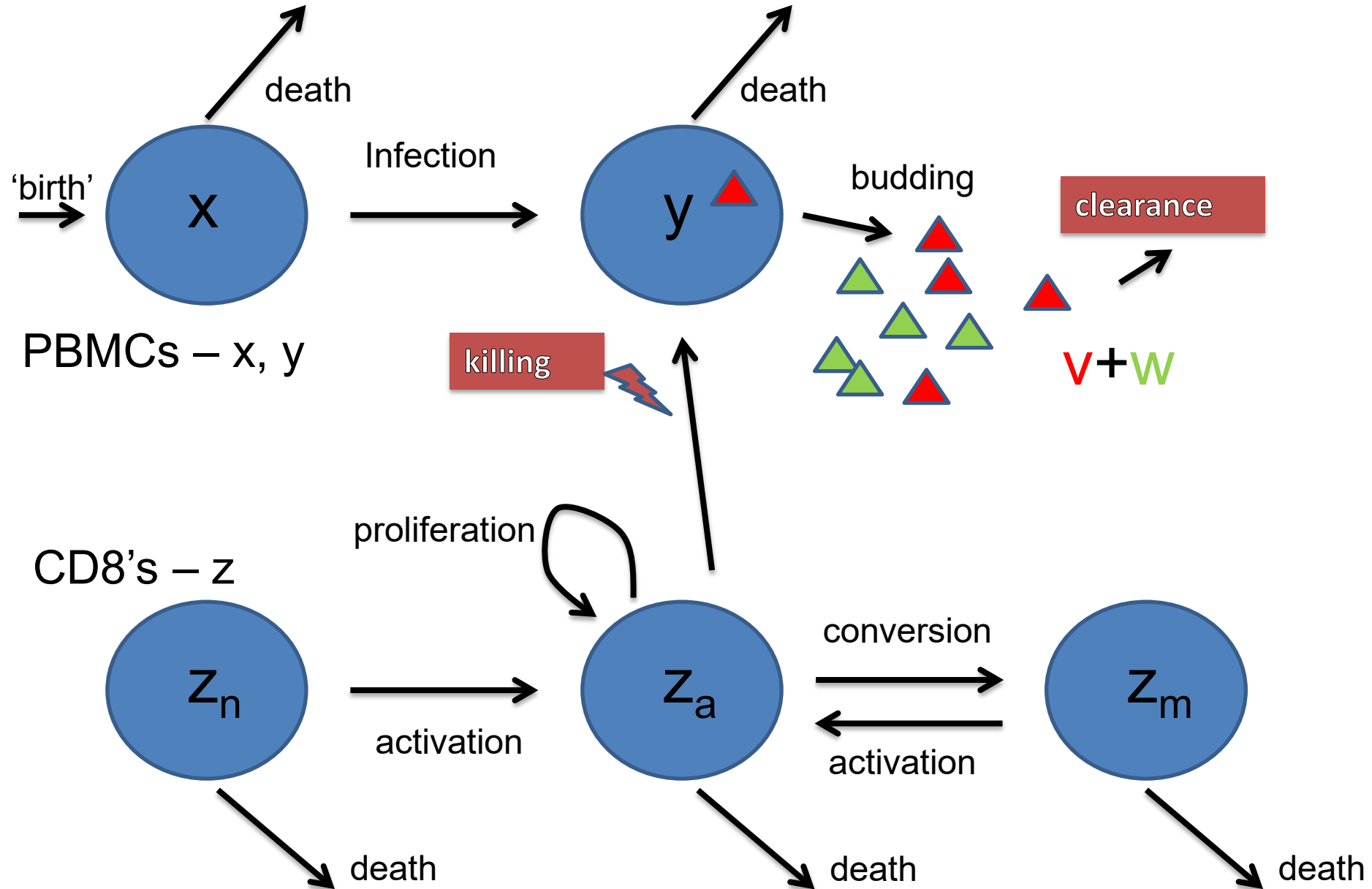
Can add in drug therapies that may excite the immune system too....

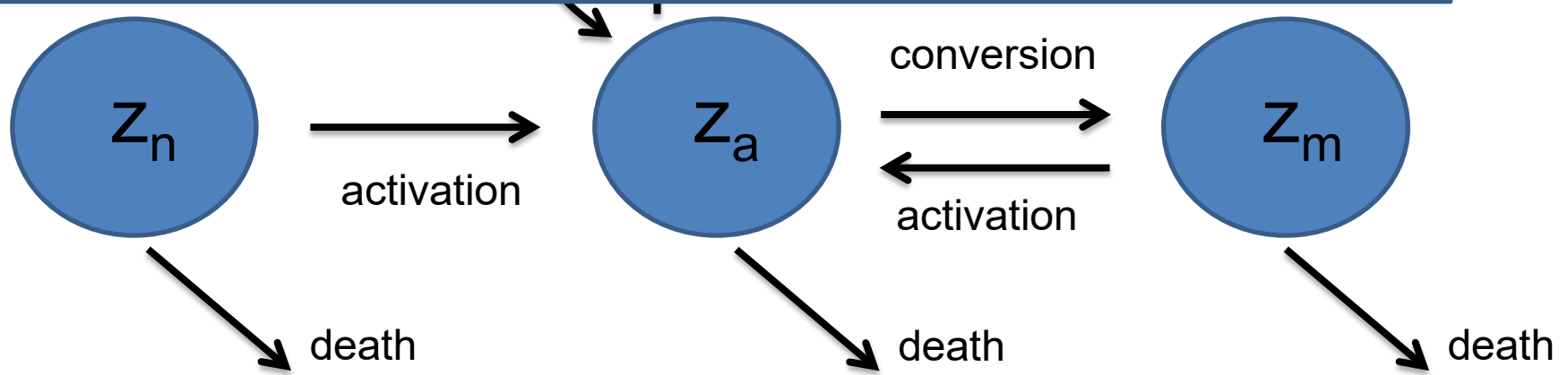
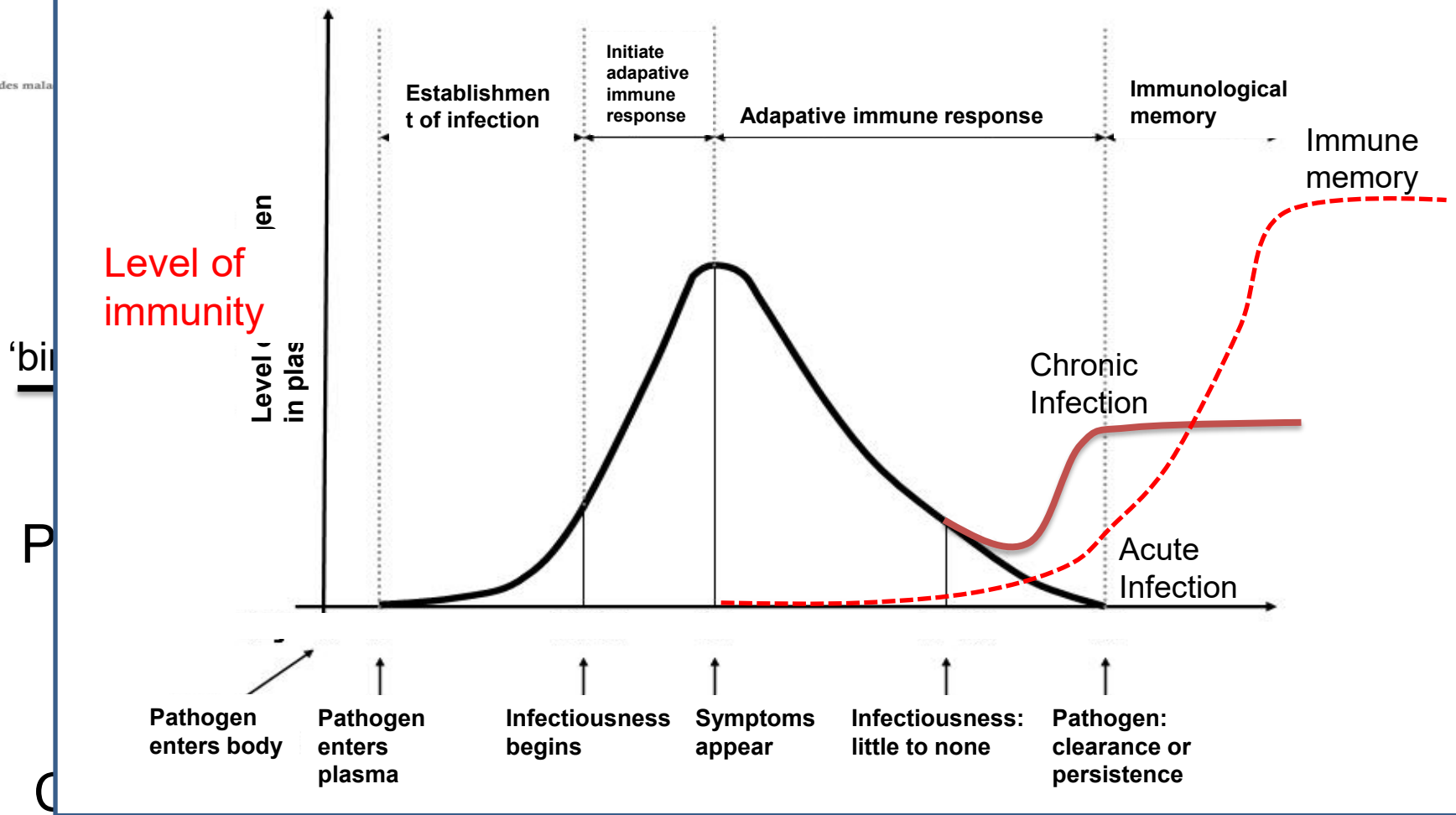
An extension



Pawelek et al (2012), Heffernan & Ciupe (2017)

Immunity/memory cells





Model-Bacteria

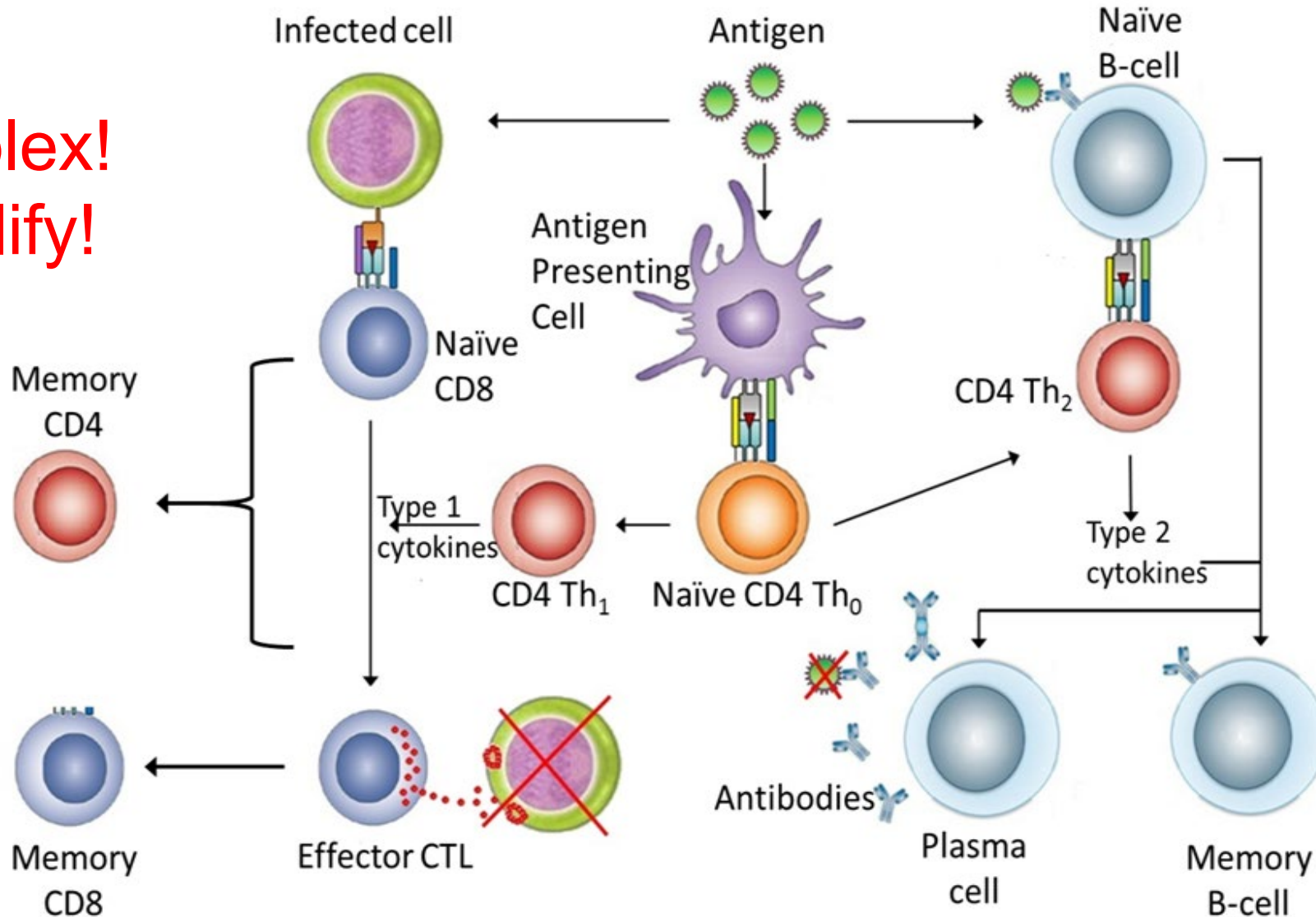
- Tuberculosis
- Macrophages
- Bacteria can replicate on their own

$$\left\{ \begin{array}{l} \frac{dM_u}{dt} = s_M - \mu_M M_u - \beta M_u B, \\ \frac{dM_i}{dt} = \beta M_u B - b M_i - \gamma M_i \frac{(T/M_i)}{(T/M_i) + c}, \\ \frac{dB}{dt} = \delta B \left(1 - \frac{B}{K}\right) + N_1 b M_i + N_2 \gamma M_i \frac{(T/M_i)}{(T/M_i) + c} - \eta M_u B - N_3 \beta M_u B, \\ \frac{dT}{dt} = s_T + \frac{c_M M_i T}{e_M T + 1} + \frac{c_B B T}{e_B T + 1} - \mu_T T. \end{array} \right.$$

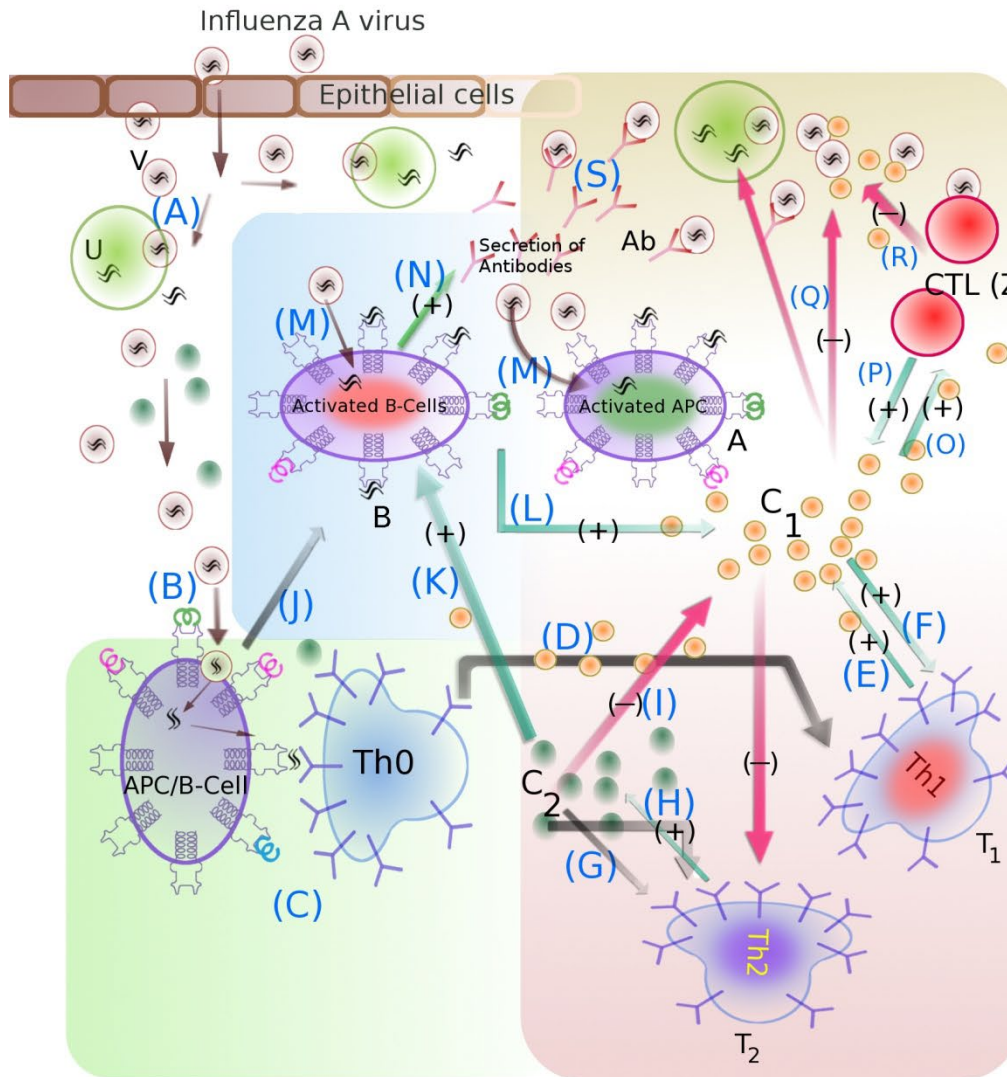
$$\mathcal{R}_0 = \frac{\delta}{(\eta + N_3 \beta) M_u^0} + \frac{N_1 b + N_2 \gamma}{b + \gamma} \frac{\beta}{\eta + N_3 \beta}$$

Understand immune system/in-host dynamics

Quite complex!
 Simplify!



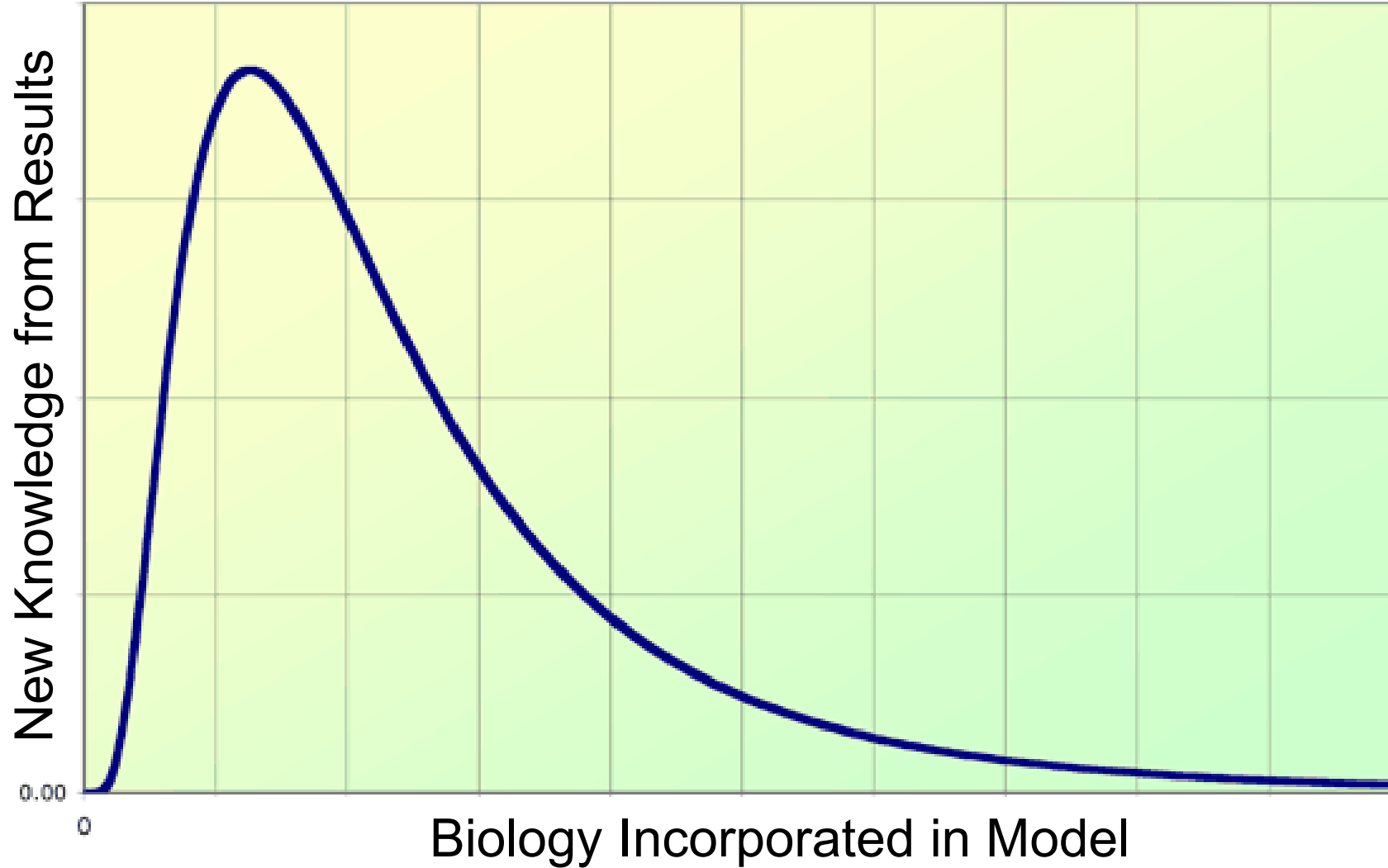
In host model: Influenza

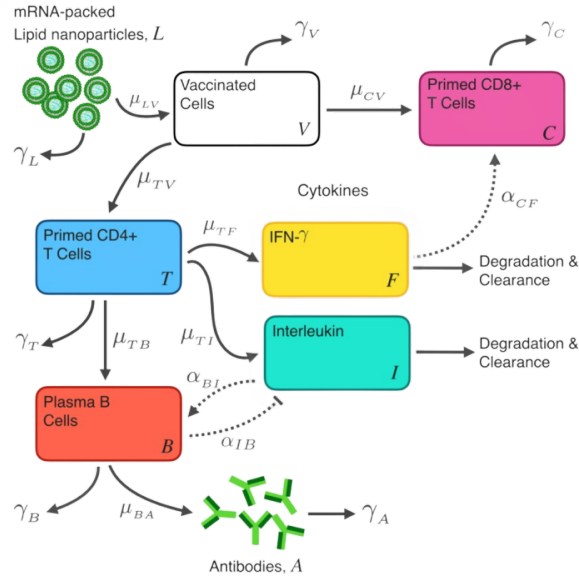


- (A): virions infect healthy cells (U).
- (B): virions are engulfed by Antigen Presenting Cells (APC) and B-Cells, are processes and a derived peptide-antigens are expressed on MHC-II surface molecule.
- (C): Naive T-cells (Th0) interact with APC/B-cell to recognize the peptide-antigens.
- (D): Class-I cytokines (Cy1) (IL-12, IFN-g, IL-18) promote Th0 differentiate into activated Th1.
- (E): Th1 up-regulates further production of Cy1 - specially IL-2, IFN-g and TFN-b.
- (F): Positive feedback loop that enhances growth of activated Th1 by Cy1.
- (G): Class-II cytokines (Cy2) promote Th0 differentiate into activated Th2.
- (H): Activated Th2 upregulates production of Cy2.
- (I): Cy2 inhibition of growth of Cy1.
- (J): APC and B-cells gets activated upon Th0 activation.
- (K): Cy2 upregulates growth and activation of B-cells.
- (L): Activated APC/B-cells up-regulate production of Cy1 (IFN-g).
- (M): Activated APC/B-cells recruit more virions for expression.
- (N): Activated B-cells secrete influenza-A specific antibodies.
- (O): Cy1 promotes activation of Cytotoxic T Lymphocytes (CTL*).
- (P): CTL* up-regulates production of Cy1.
- (Q): Cy1 inhibition of viral mRNA (editing).
- (R): CTL* mediated direct lysis of infected cells and virions.
- (S): B-cell secreted antibody mediated lysis of infected cells and virions.

MANY EQUATIONS, ASSUMPTIONS – PARAMETER IDENTIFIABILITY!!

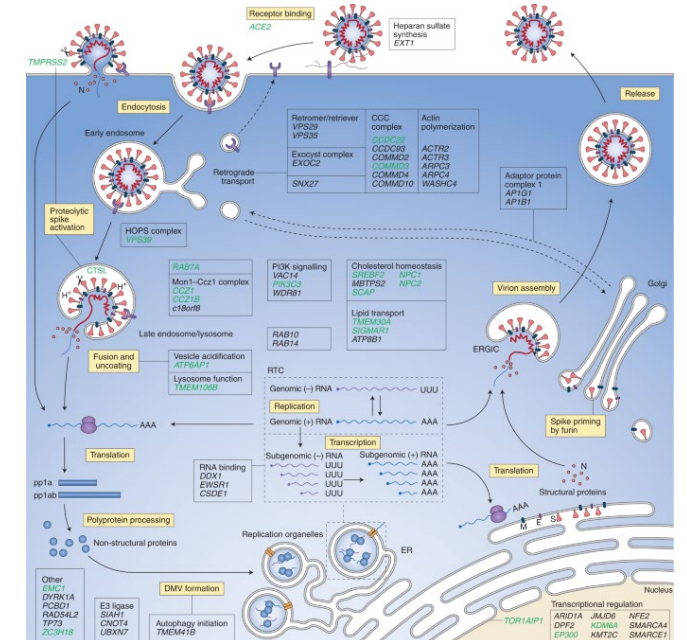
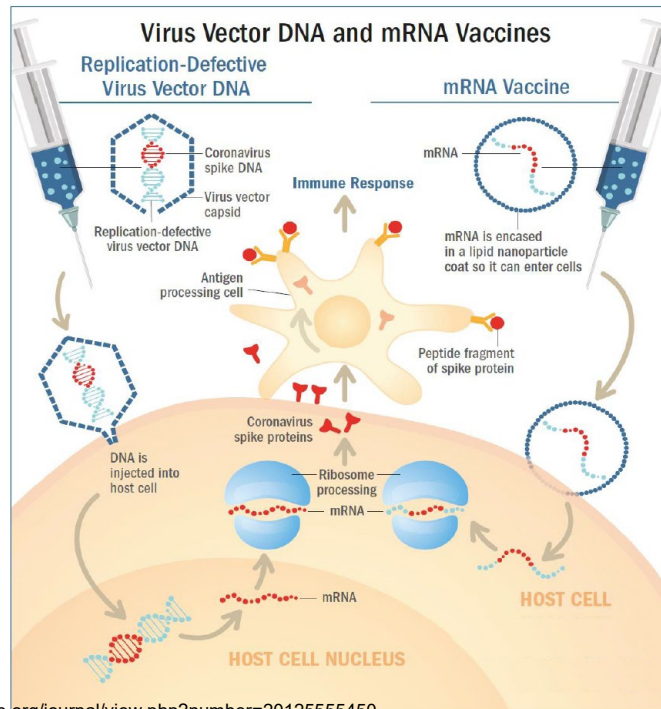
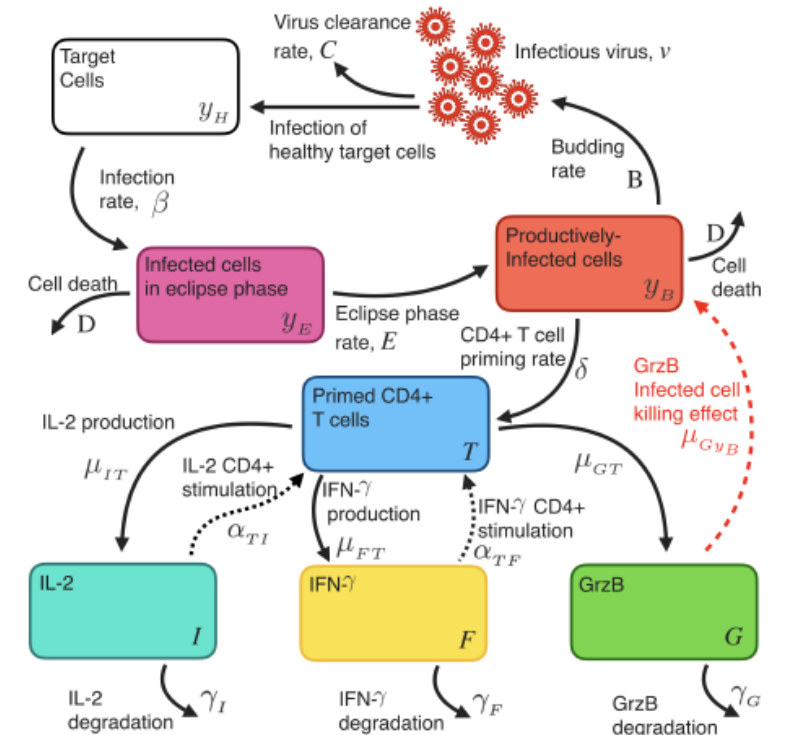
More vs. More





In-host model

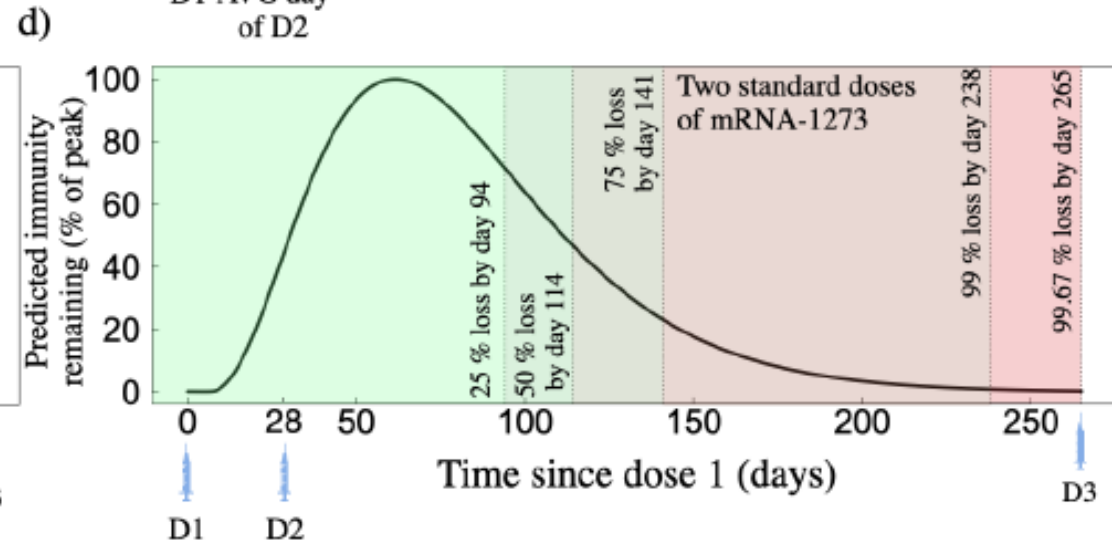
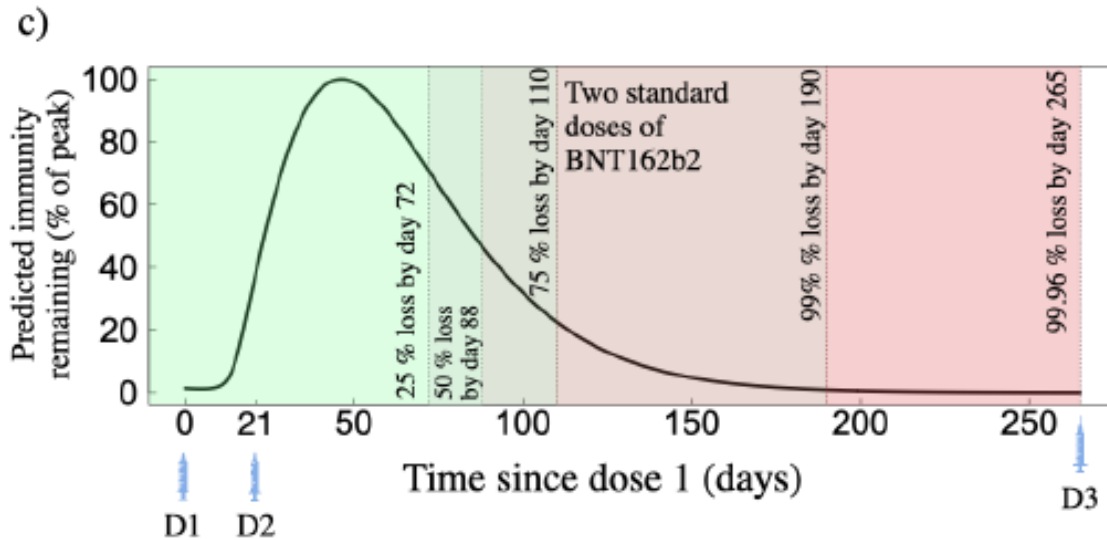
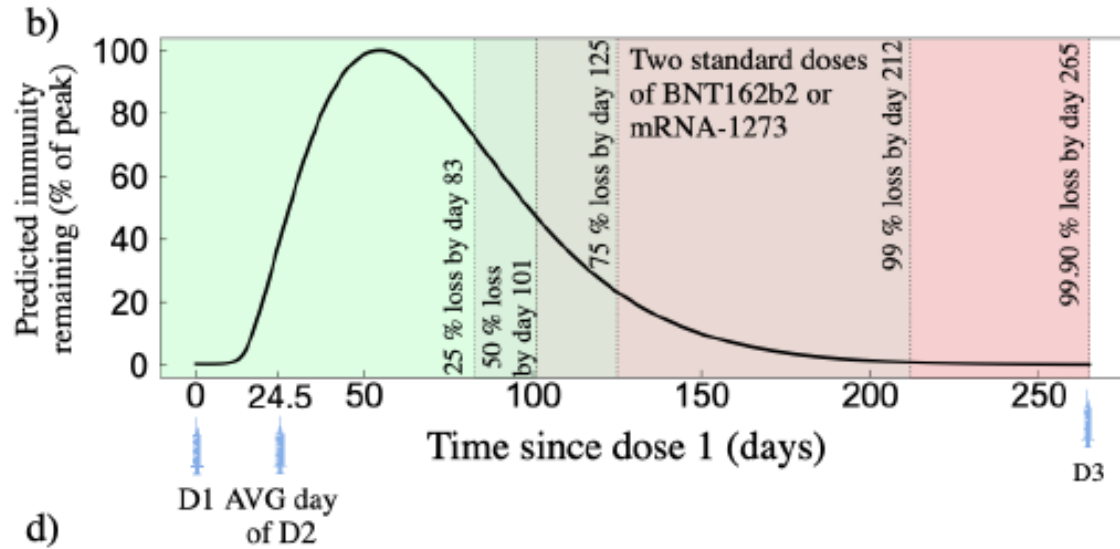
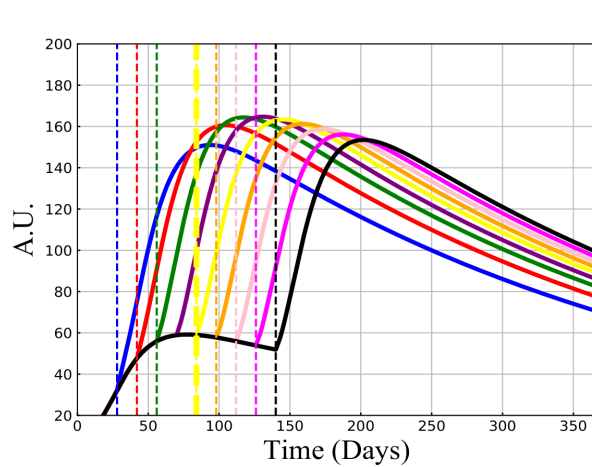
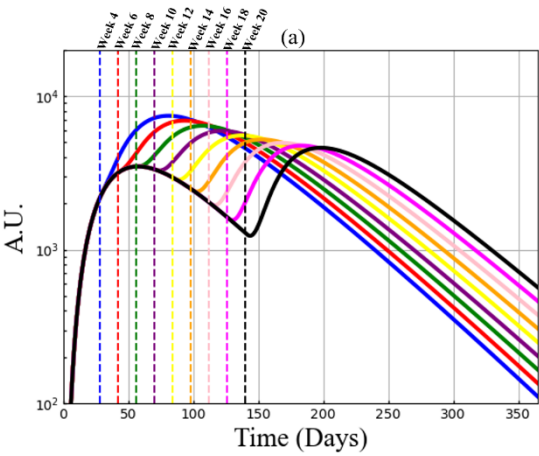
Farhang-Sardroodi et al., (2021) Vaccines
 Korosec et al., (2022) Scientific Reports
 Gholami et al., (2022) Math Biosci
 Lin, Korosec, et al., (2022) J Am Soc Microbiol



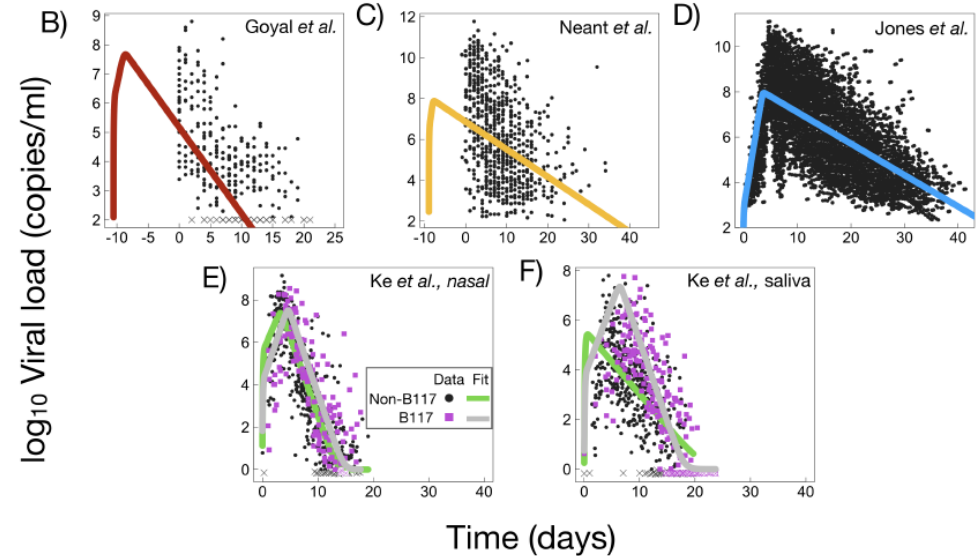
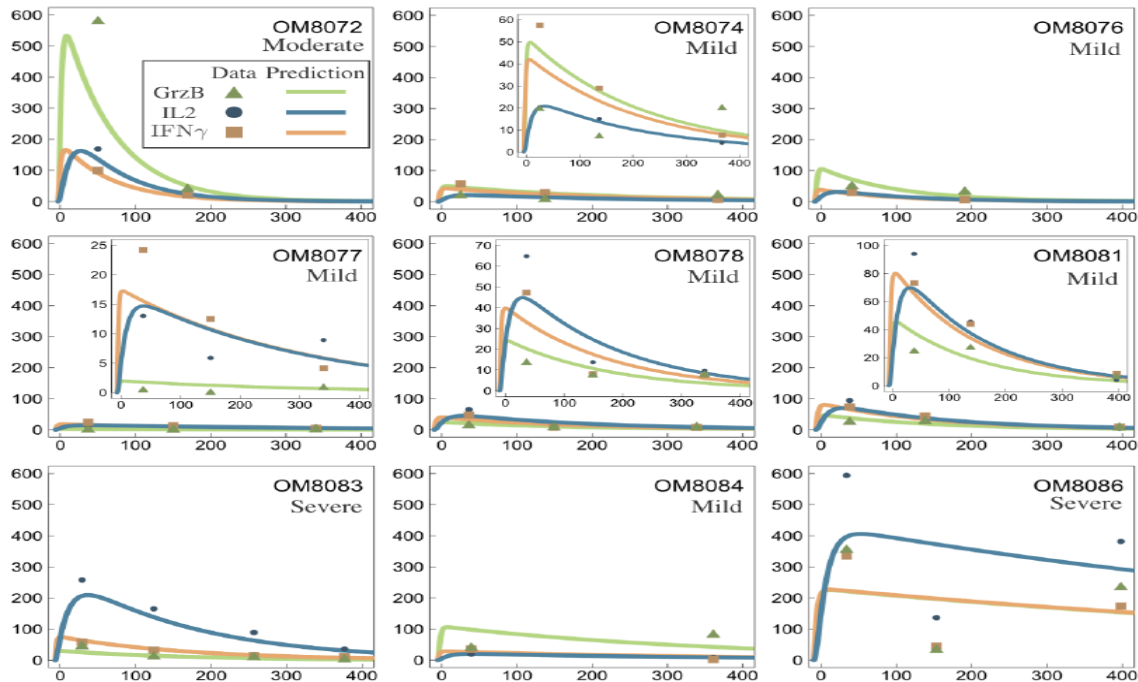
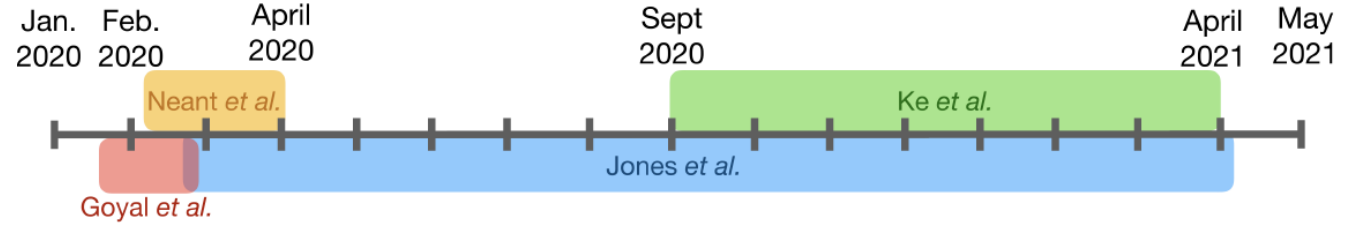
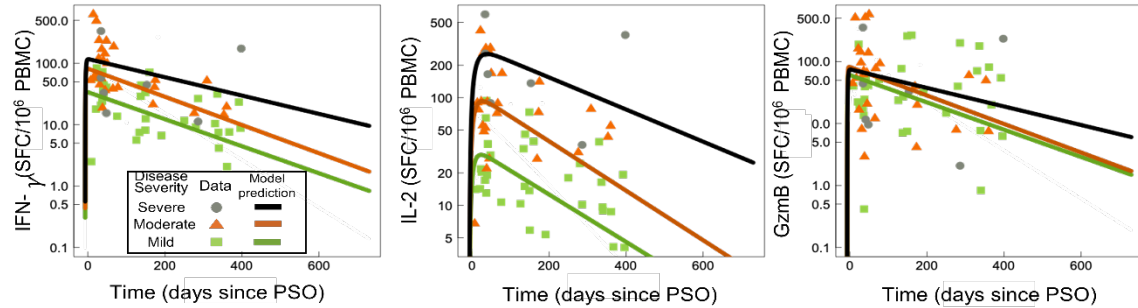
Modelling Vaccination In-Host

Antibodies

T-cells

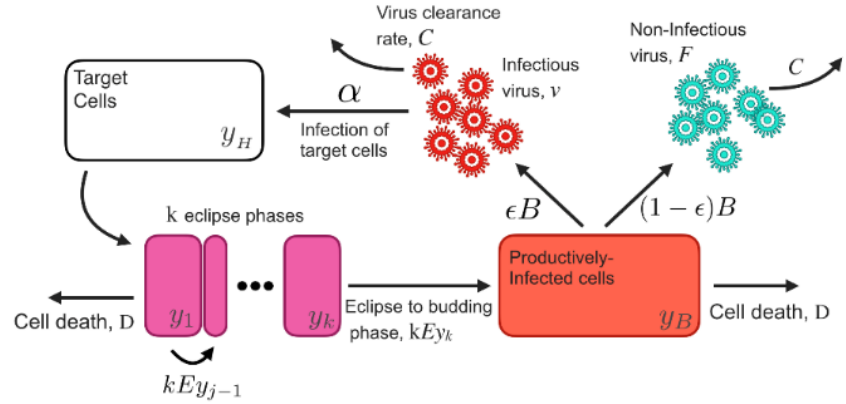


Modelling Infection In-Host

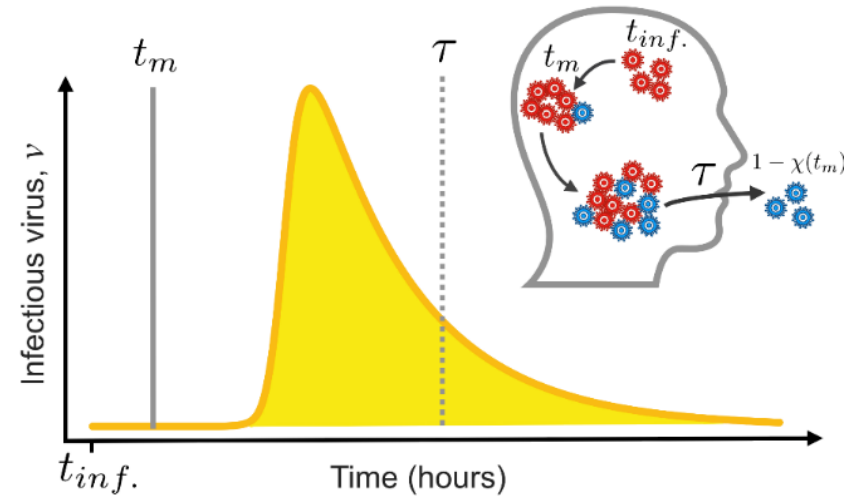


Severity of Infection, Mutation

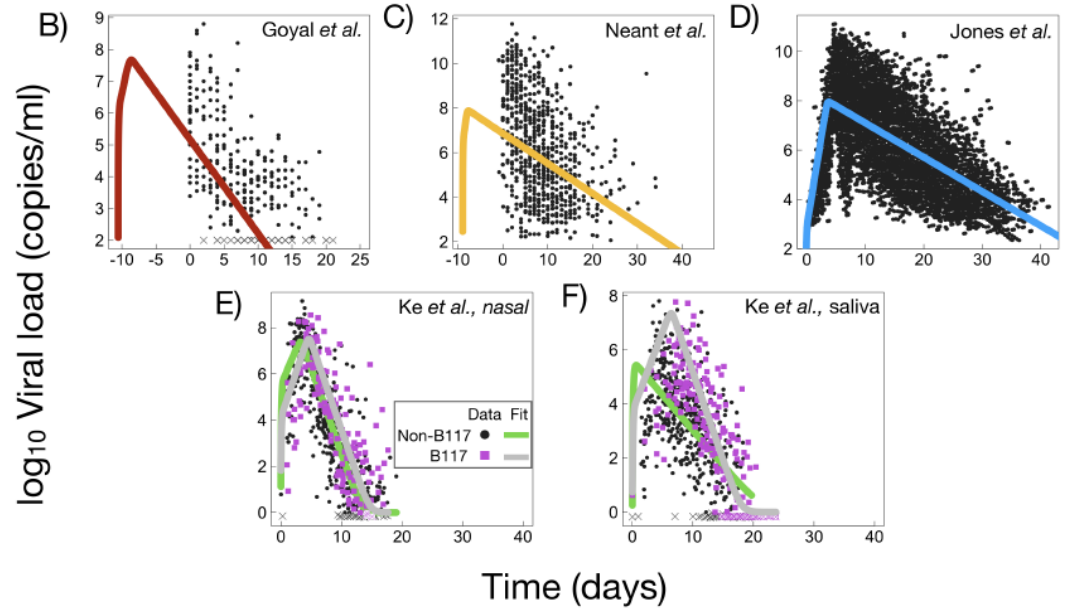
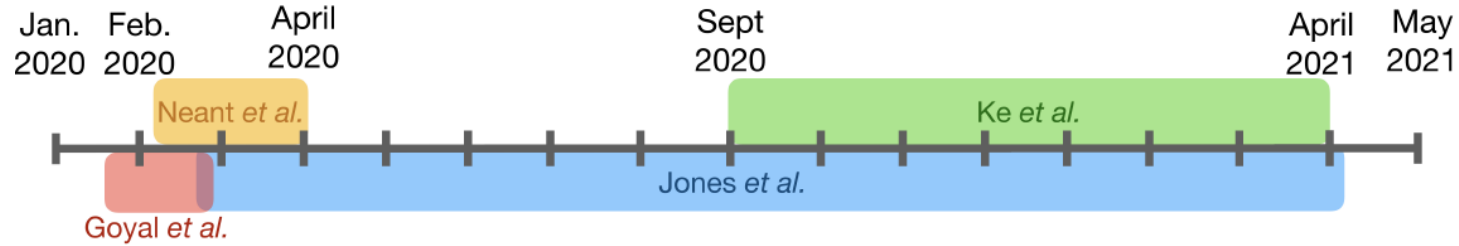
A) Schematic of target-cell limited model (Eq.1)



B) Schematic of transmission bottleneck model (Eq.2)



$t_{inf.}$: Time of infection
 t_m : Time of virus mutation, $0 < t_m < \tau$
 τ : Time of disease transmission
 $1 - \chi(t_m)$: Survival probability of mutated strain



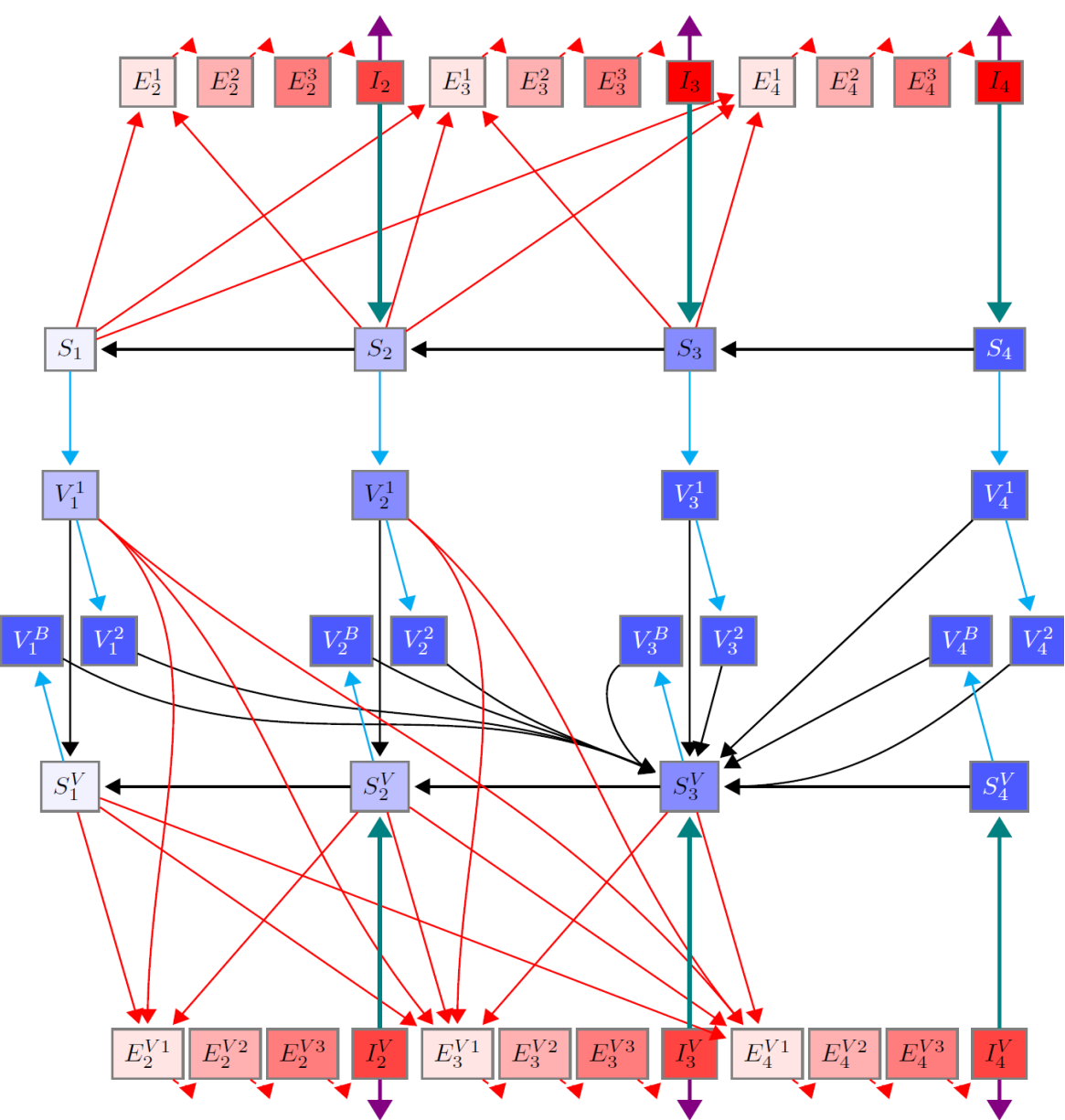
Infection & Immunity Observations

Model Fitting

- Immunity gained after infection correlates with the severity of the infection
- Immunity gained from infection and vaccination can wane over similar periods of time
- Mild infections are shorter in duration than moderate infections
- Moderate infections are shorter than severe infections
- Viral load correlates with severity of infection, and thus will affect transmissibility

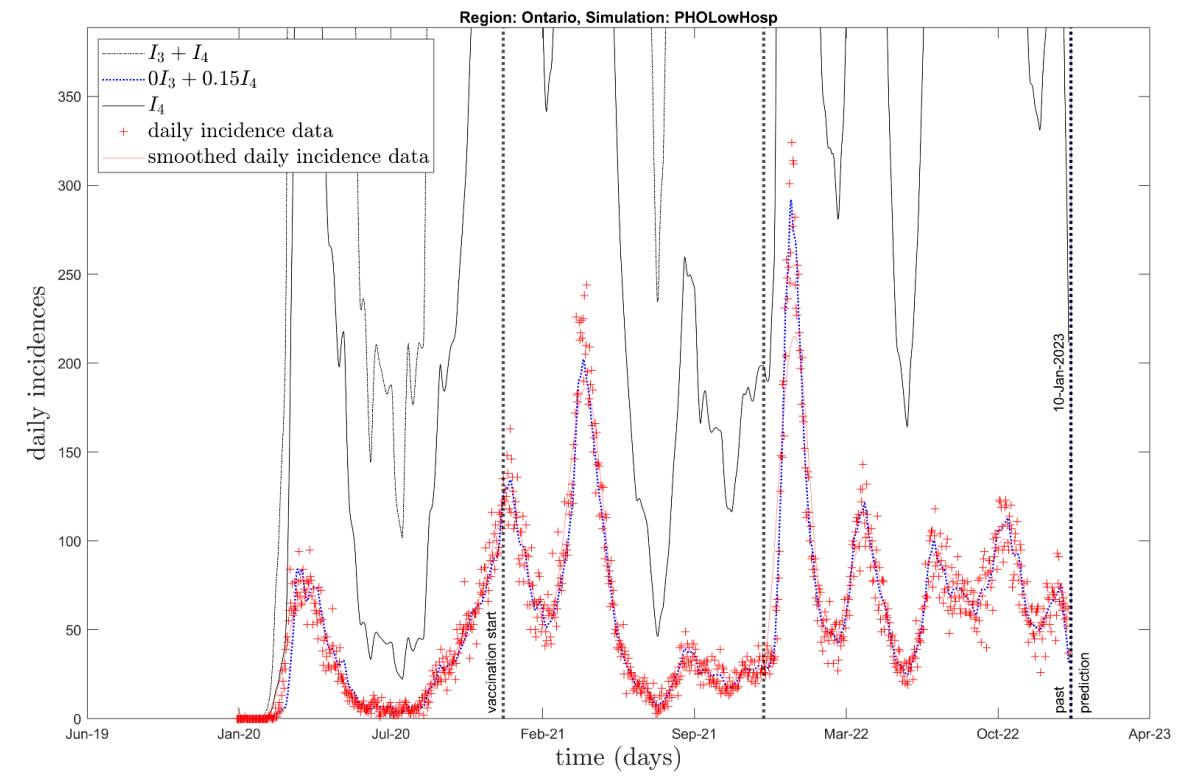
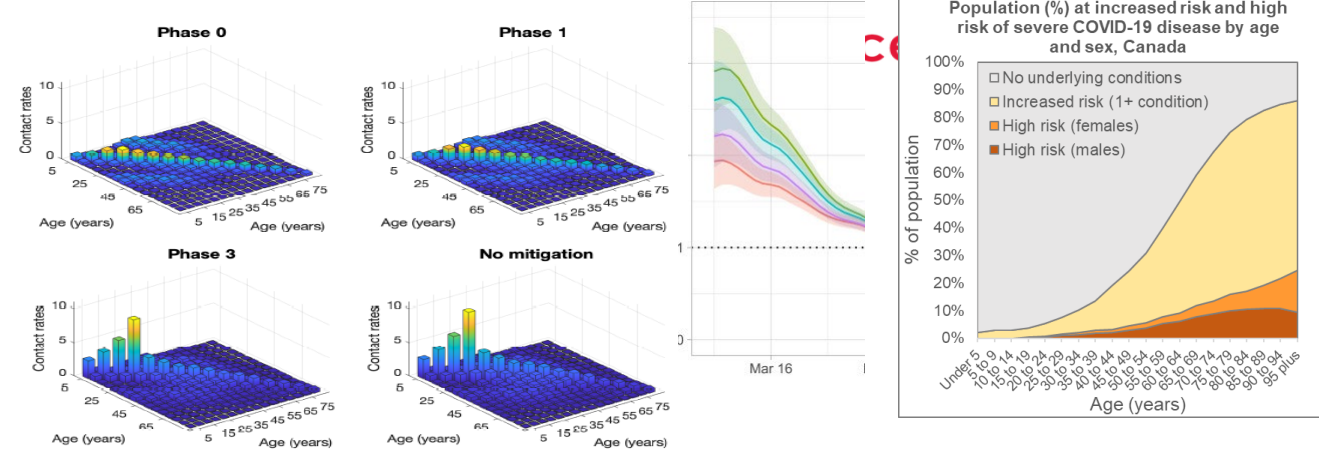
Extra

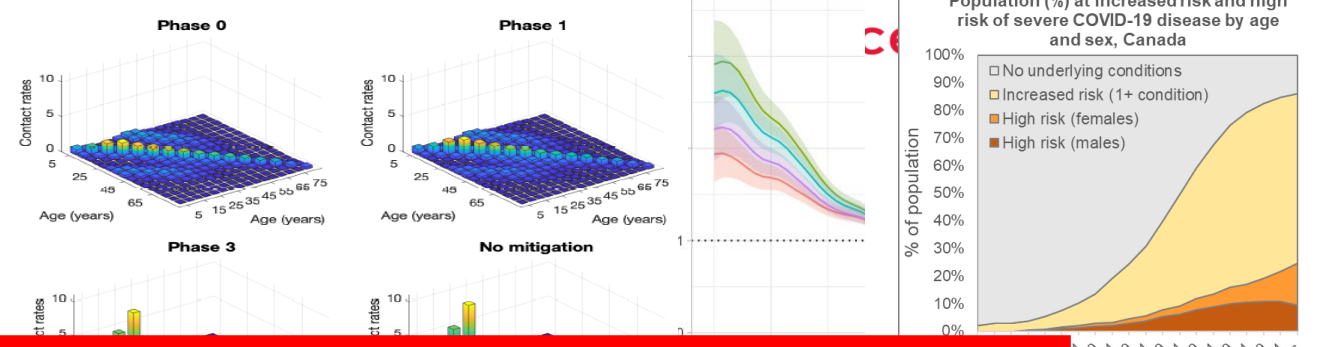
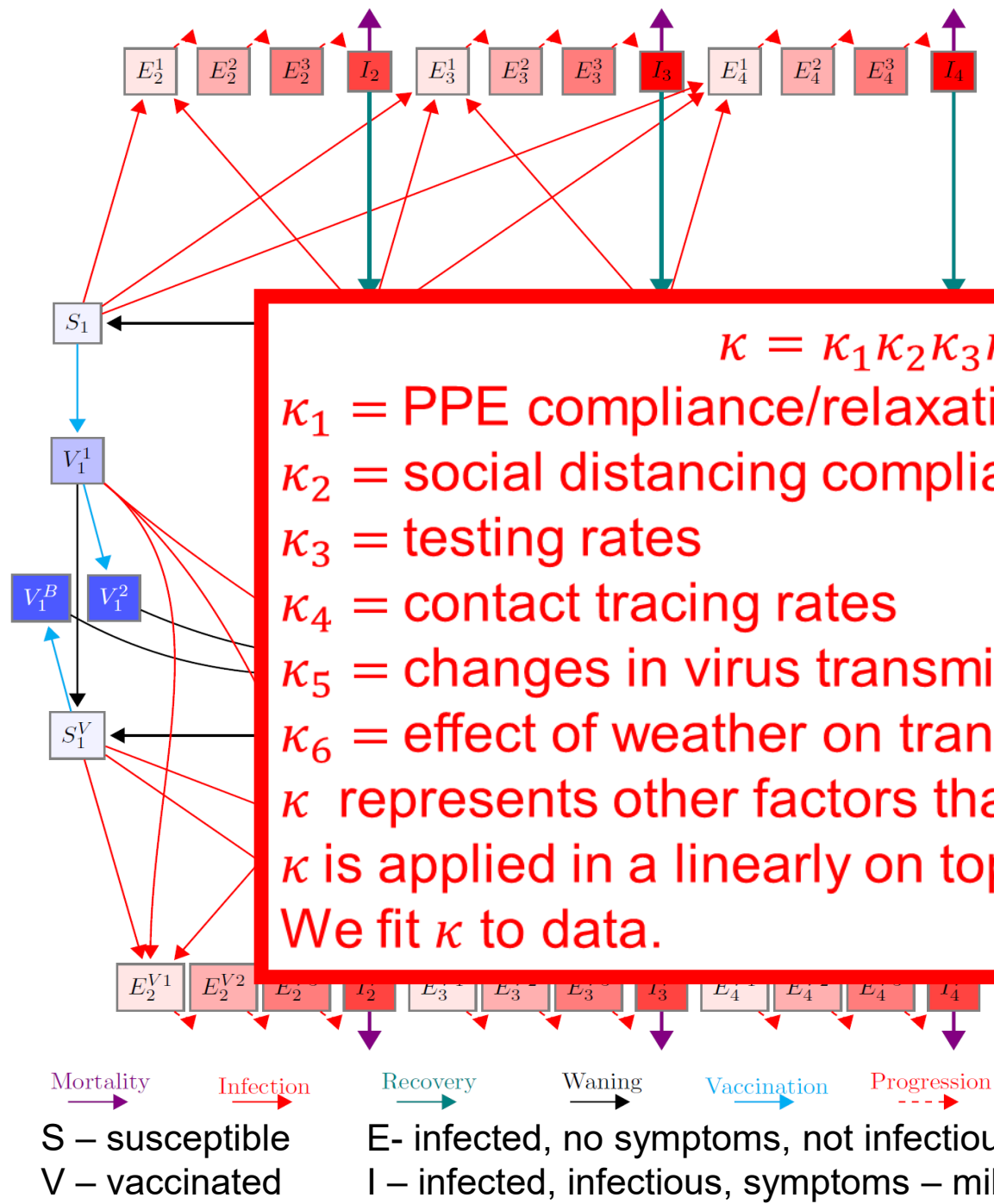
- Literature
- Assumptions
- Sensitivity analysis



→ Mortality
 → Infection
 → Recovery
 → Waning
 → Vaccination
 → Progression

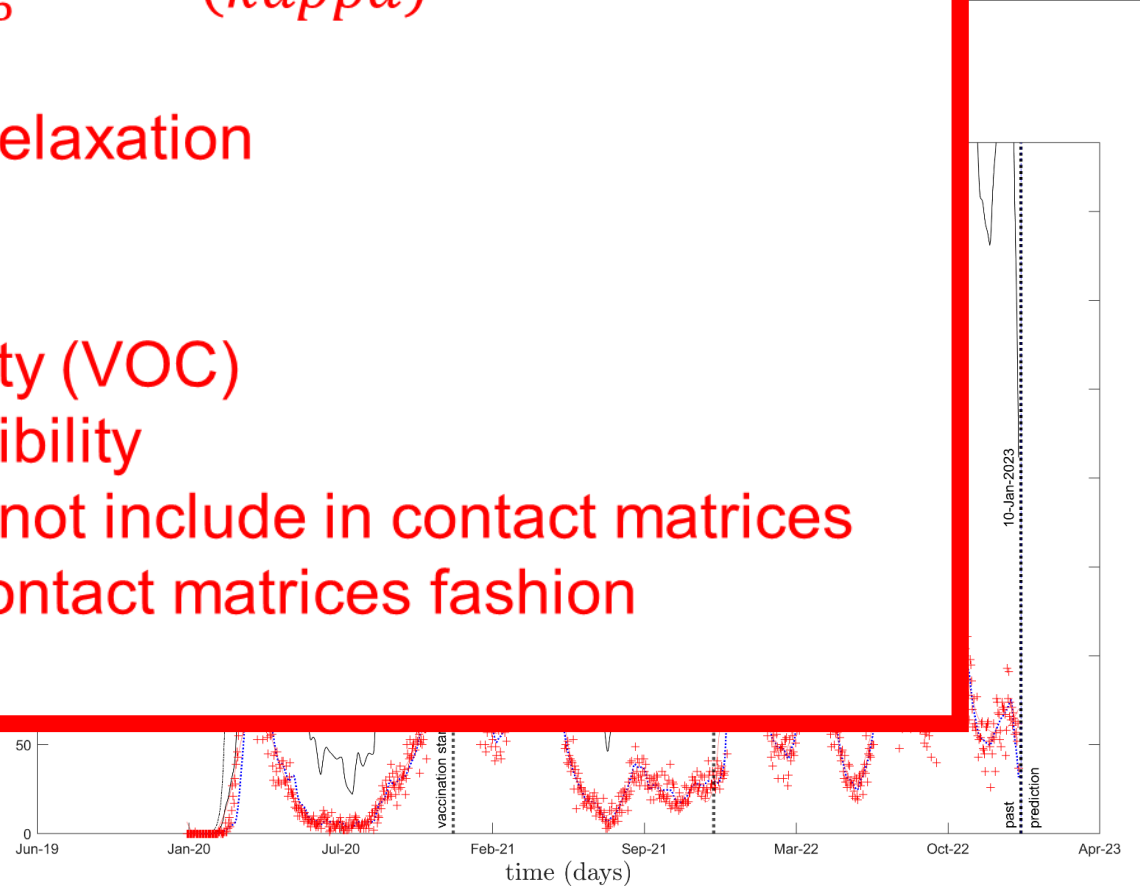
S – susceptible E- infected, no symptoms, not infectious
 V – vaccinated I – infected, infectious, symptoms – mild (asymptomatic), moderate, severe





$\kappa = \kappa_1 \kappa_2 \kappa_3 \kappa_4 \kappa_5 \kappa_6$ (kappa)

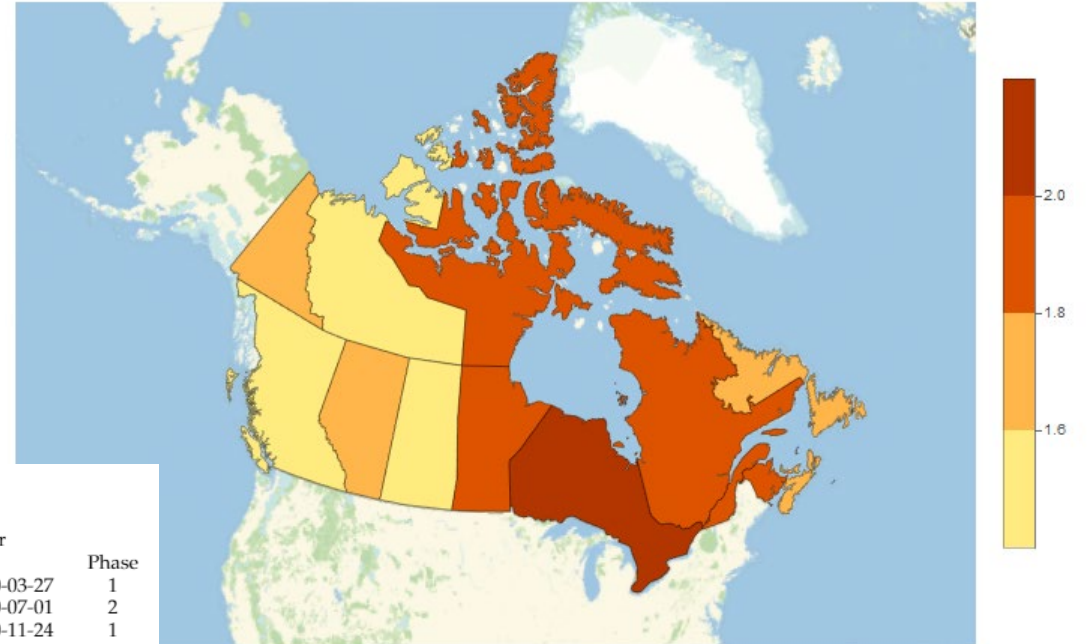
κ_1 = PPE compliance/relaxation
 κ_2 = social distancing compliance/relaxation
 κ_3 = testing rates
 κ_4 = contact tracing rates
 κ_5 = changes in virus transmissibility (VOC)
 κ_6 = effect of weather on transmissibility
 κ represents other factors that are not include in contact matrices
 κ is applied in a linearly on top of contact matrices fashion
 We fit κ to data.



Analysis: Public Health Mitigation

Scale	School	Work
0	no restrictions	no restrictions
1	open with restrictions	open with restrictions
2	blended learning	most businesses closed except for some sectors
3	closed	only essential services operating

Scale	Other
0	no restrictions
1	minor restrictions on gatherings, travelling, activities
2	moderate restrictions on gatherings, travelling, activities
3	strict restrictions on gatherings, travelling, activities



Start	End	Phase
2020-03-05	2020-05-14	3
2020-05-14	2020-09-01	2
2020-09-01	2020-11-30	1
2020-11-30	2020-12-18	2
2020-12-18	2021-01-11	3
2021-01-11	2021-03-27	1
2021-03-27	2021-04-06	3
2021-04-06	2021-05-07	1
2021-05-07	2021-05-25	3
2021-05-25	2021-06-30	1
2021-06-30	2021-07-28	3

Start	End	Phase
2020-03-17	2020-03-28	1
2020-03-28	2020-05-04	3
2020-05-04	2020-12-13	2
2020-12-13	2021-02-08	3
2020-02-08	2021-05-04	2
2021-05-04	2021-06-10	3
2021-06-10	2021-07-01	1
2021-07-01	2021-07-28	0

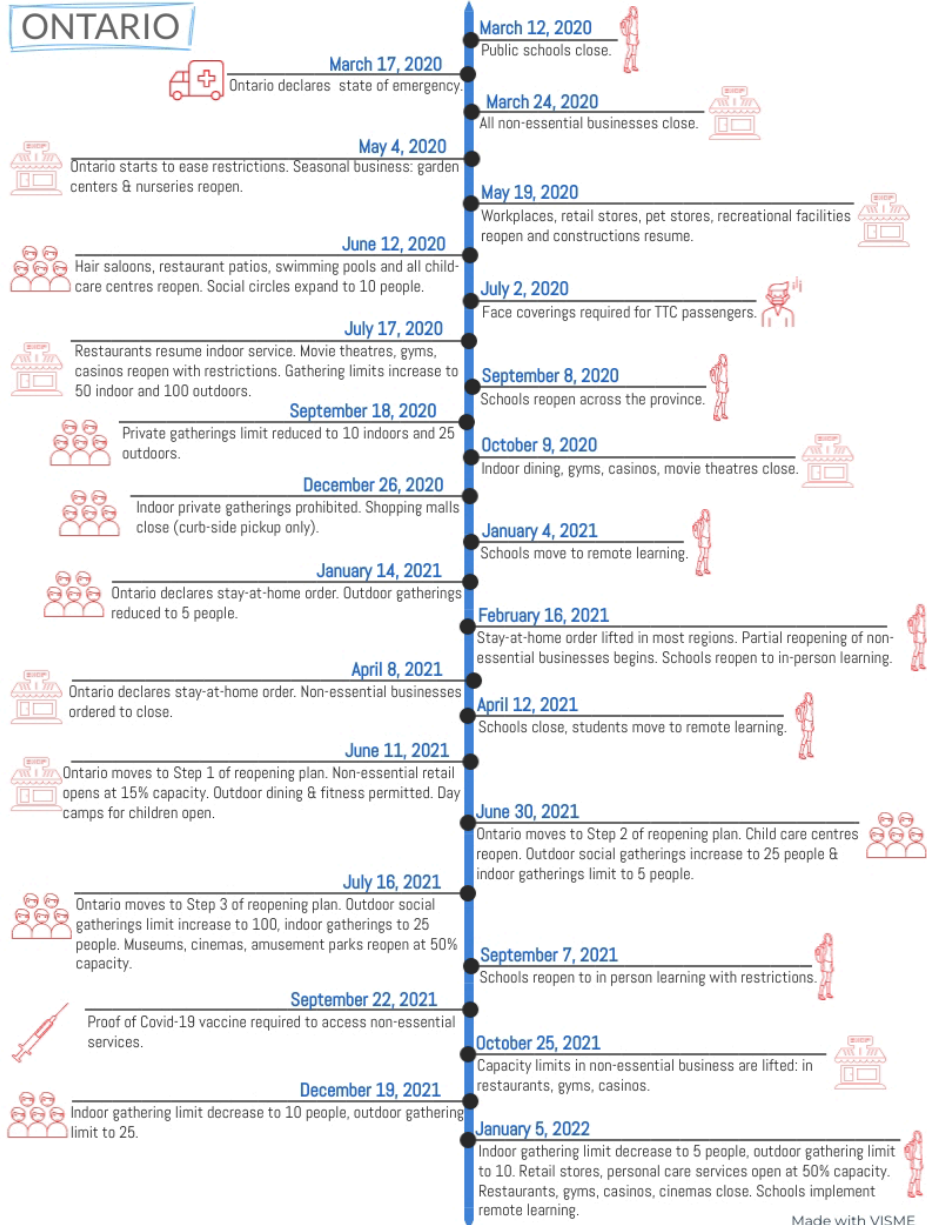
Start	End	Phase
2020-03-12	2020-03-27	1
2020-03-27	2020-07-01	2
2020-07-01	2020-11-24	1
2020-11-24	2021-06-10	3
2020-06-10	2021-07-01	2
2021-07-01	2021-07-28	0

Start	End	Phase
2020-03-23	2020-09-08	3
2020-09-08	2020-10-26	1

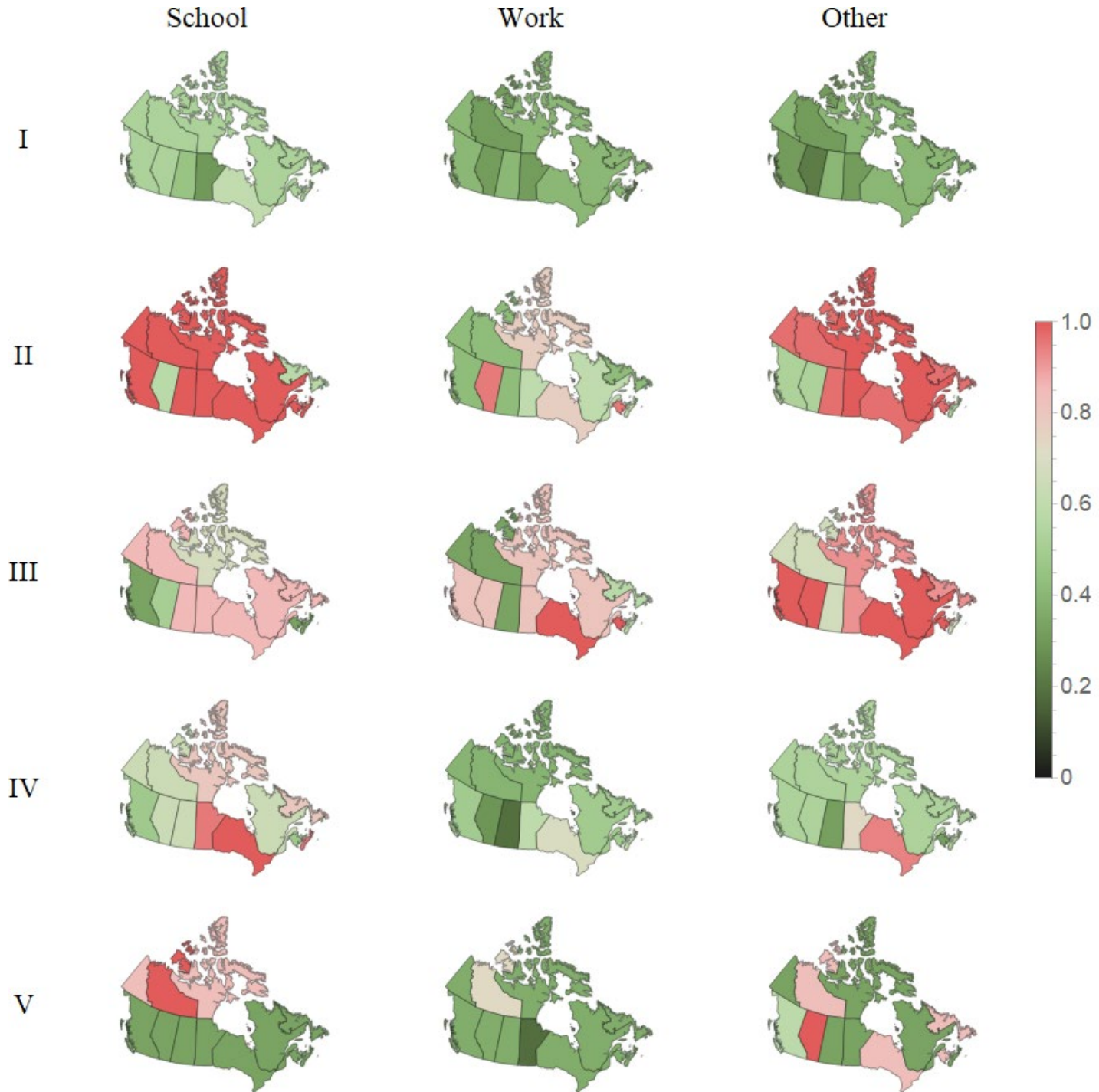
Start	End	Phase
2020-03-20	2020-04-01	1
2020-04-01	2020-05-04	3
2020-05-04	2020-06-01	2
2020-06-01	2021-11-02	1
2020-11-02	2020-11-12	0

Start	End	Phase
2020-03-17	2020-03-20	1
2020-03-20	2020-04-01	2
2020-04-01	2020-06-01	3
2020-06-01	2021-09-28	2

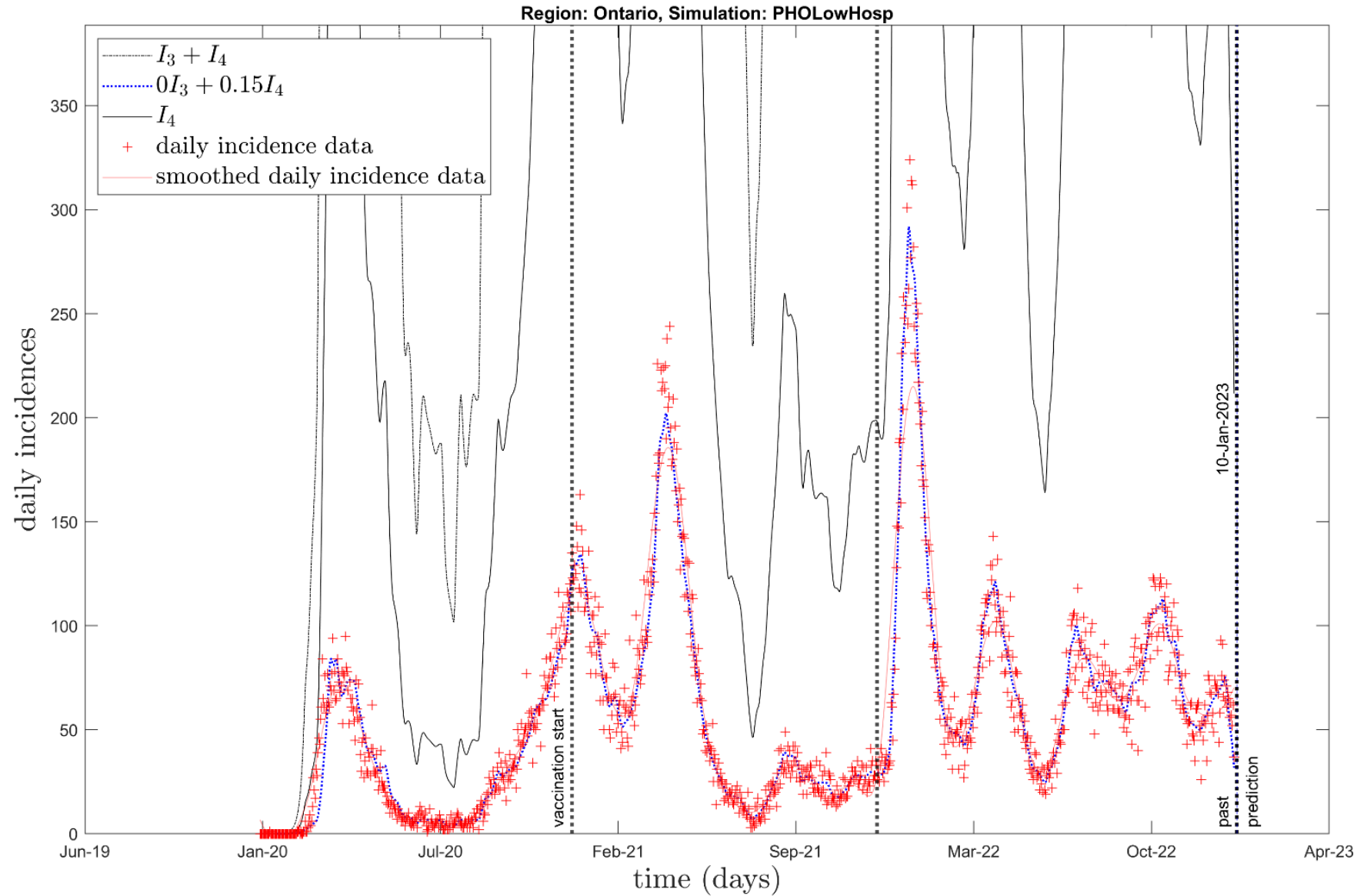
ONTARIO



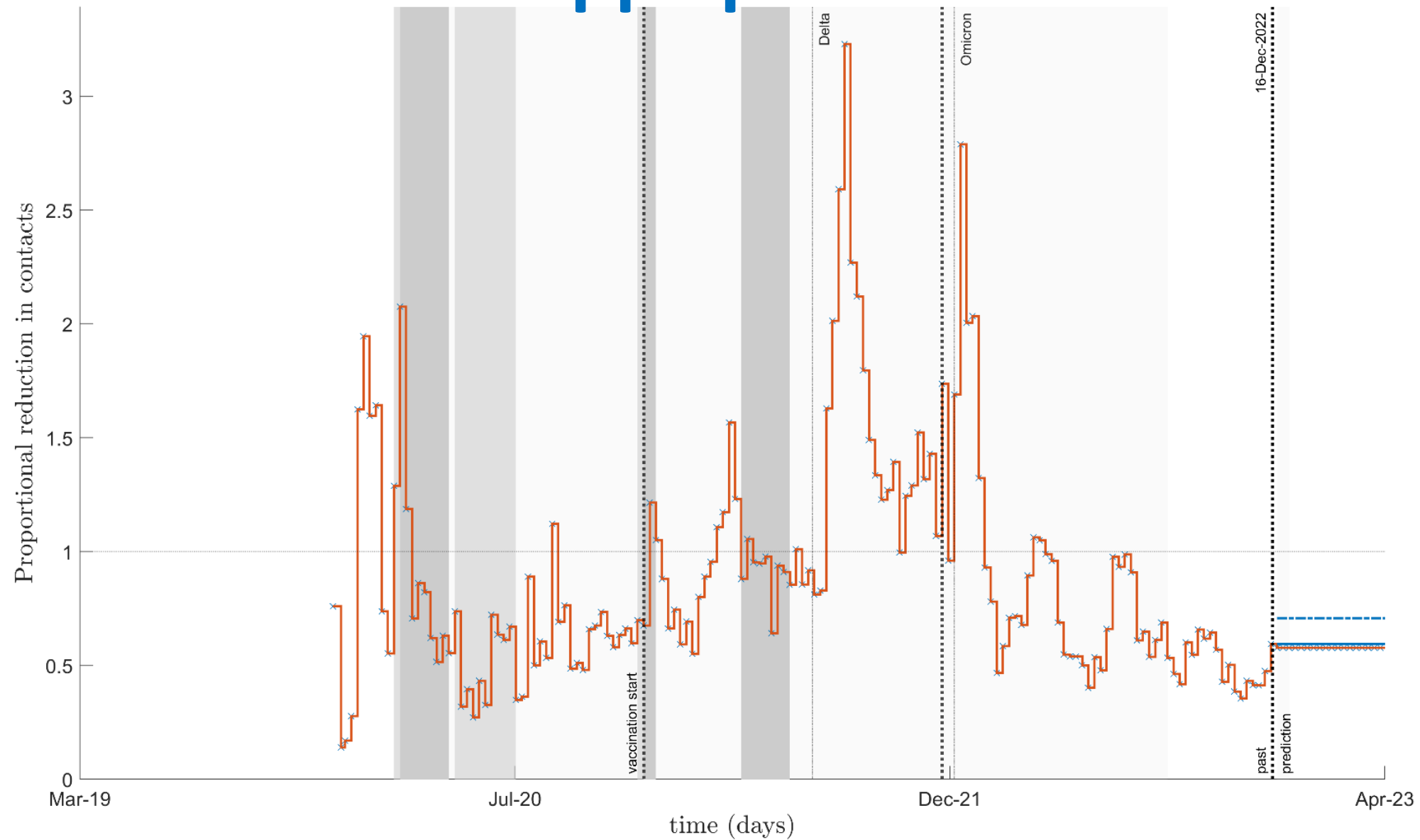
Made with VISME

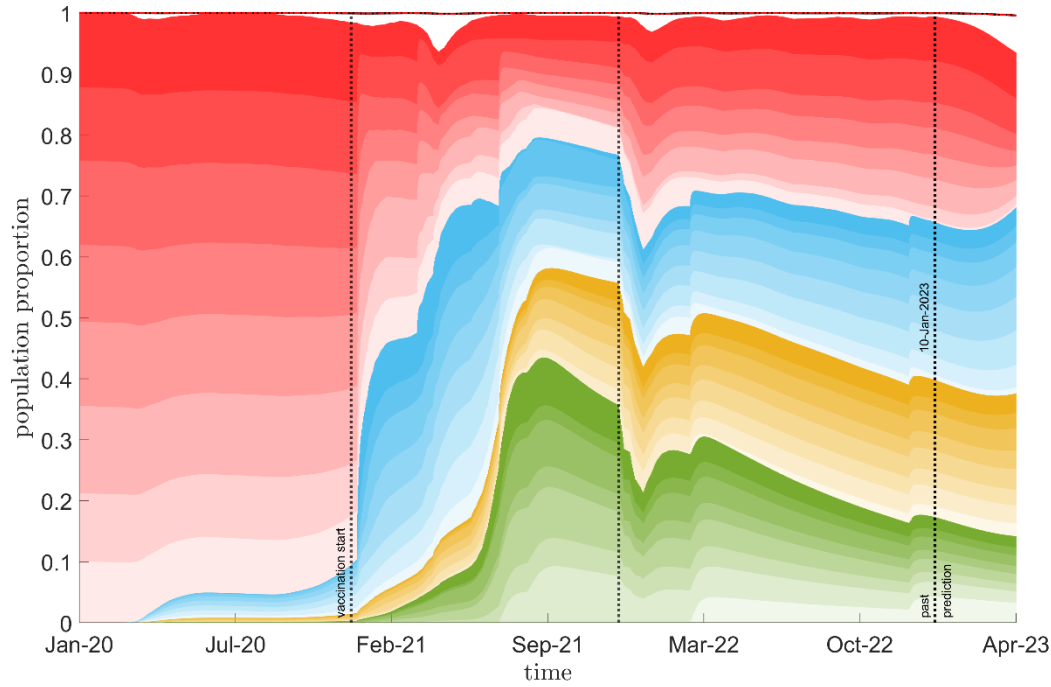


Calibration to Hosp Data



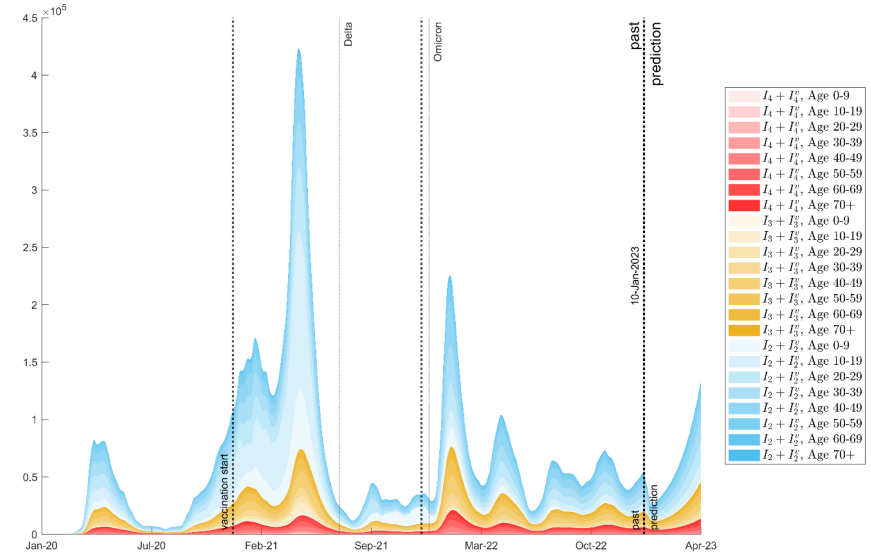
Kappa parameter



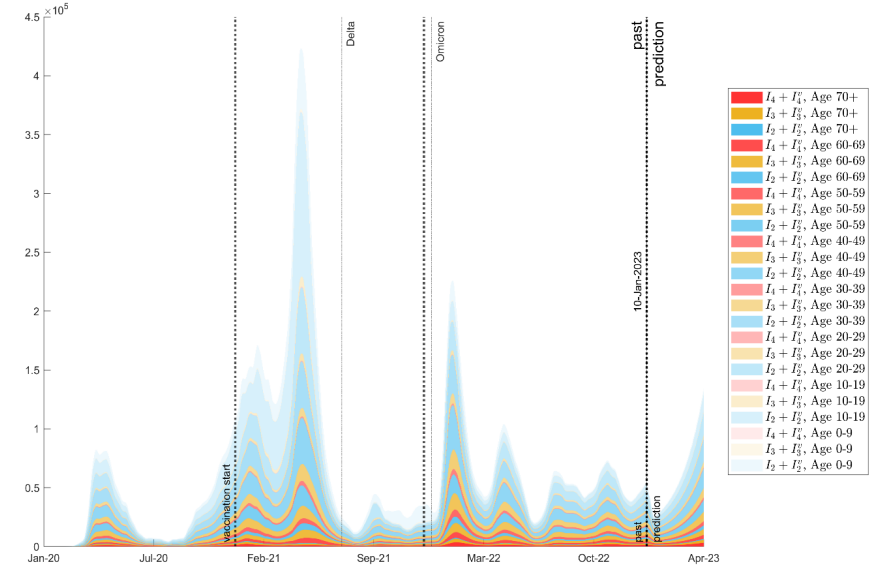


- $S_4 + S_4^V + \sum_{i=3}^4 V_i^1 + \sum_{i=1}^4 V_i^2 + \sum_{i=1}^4 V_i^B$, Age 0-9
- $S_4 + S_4^V + \sum_{i=3}^4 V_i^1 + \sum_{i=1}^4 V_i^2 + \sum_{i=1}^4 V_i^B$, Age 10-19
- $S_4 + S_4^V + \sum_{i=3}^4 V_i^1 + \sum_{i=1}^4 V_i^2 + \sum_{i=1}^4 V_i^B$, Age 20-29
- $S_4 + S_4^V + \sum_{i=3}^4 V_i^1 + \sum_{i=1}^4 V_i^2 + \sum_{i=1}^4 V_i^B$, Age 30-39
- $S_4 + S_4^V + \sum_{i=3}^4 V_i^1 + \sum_{i=1}^4 V_i^2 + \sum_{i=1}^4 V_i^B$, Age 40-49
- $S_4 + S_4^V + \sum_{i=3}^4 V_i^1 + \sum_{i=1}^4 V_i^2 + \sum_{i=1}^4 V_i^B$, Age 50-59
- $S_4 + S_4^V + \sum_{i=3}^4 V_i^1 + \sum_{i=1}^4 V_i^2 + \sum_{i=1}^4 V_i^B$, Age 60-69
- $S_4 + S_4^V + \sum_{i=3}^4 V_i^1 + \sum_{i=1}^4 V_i^2 + \sum_{i=1}^4 V_i^B$, Age 70+
- $S_3 + S_3^V + V_2^1$, Age 0-9
- $S_3 + S_3^V + V_2^1$, Age 10-19
- $S_3 + S_3^V + V_2^1$, Age 20-29
- $S_3 + S_3^V + V_2^1$, Age 30-39
- $S_3 + S_3^V + V_2^1$, Age 40-49
- $S_3 + S_3^V + V_2^1$, Age 50-59
- $S_3 + S_3^V + V_2^1$, Age 60-69
- $S_3 + S_3^V + V_2^1$, Age 70+

- $S_2 + S_2^V + V_1^1$, Age 0-9
- $S_2 + S_2^V + V_1^1$, Age 10-19
- $S_2 + S_2^V + V_1^1$, Age 20-29
- $S_2 + S_2^V + V_1^1$, Age 30-39
- $S_2 + S_2^V + V_1^1$, Age 40-49
- $S_2 + S_2^V + V_1^1$, Age 50-59
- $S_2 + S_2^V + V_1^1$, Age 60-69
- $S_2 + S_2^V + V_1^1$, Age 70+
- $S_1 + S_1^V$, Age 0-9
- $S_1 + S_1^V$, Age 10-19
- $S_1 + S_1^V$, Age 20-29
- $S_1 + S_1^V$, Age 30-39
- $S_1 + S_1^V$, Age 40-49
- $S_1 + S_1^V$, Age 50-59
- $S_1 + S_1^V$, Age 60-69
- $S_1 + S_1^V$, Age 70+
- vaccination start
- booster vaccination start
- past/prediction

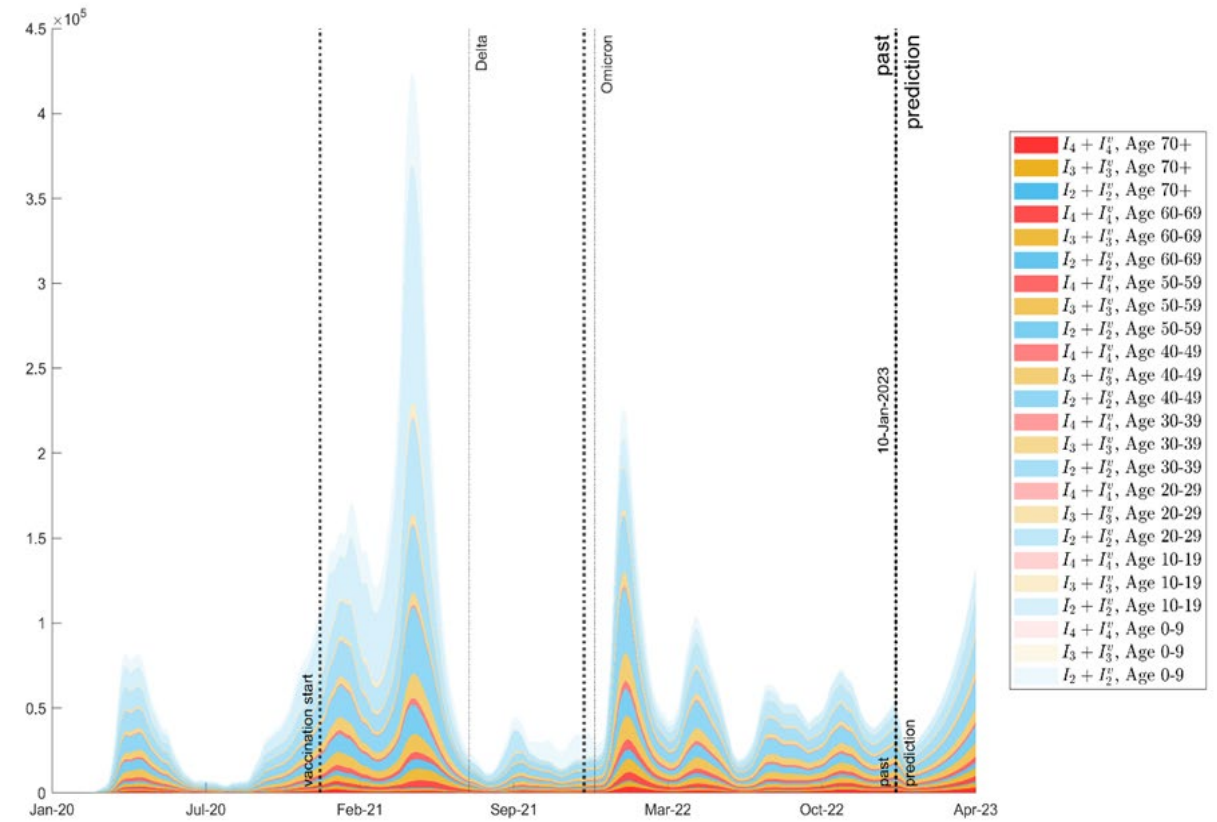


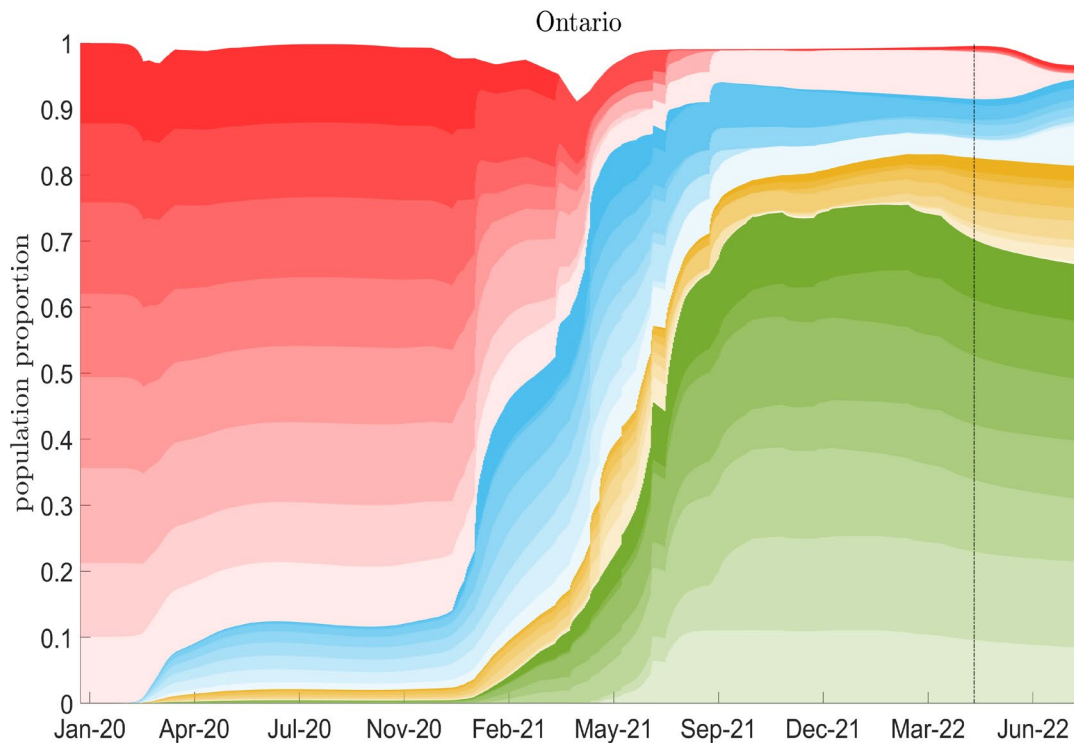
- $I_4 + I_4^1$, Age 0-9
- $I_4 + I_4^1$, Age 10-19
- $I_4 + I_4^1$, Age 20-29
- $I_4 + I_4^1$, Age 30-39
- $I_4 + I_4^1$, Age 40-49
- $I_4 + I_4^1$, Age 50-59
- $I_4 + I_4^1$, Age 60-69
- $I_4 + I_4^1$, Age 70+
- $I_3 + I_3^1$, Age 0-9
- $I_3 + I_3^1$, Age 10-19
- $I_3 + I_3^1$, Age 20-29
- $I_3 + I_3^1$, Age 30-39
- $I_3 + I_3^1$, Age 40-49
- $I_3 + I_3^1$, Age 50-59
- $I_3 + I_3^1$, Age 60-69
- $I_3 + I_3^1$, Age 70+
- $I_2 + I_2^1$, Age 0-9
- $I_2 + I_2^1$, Age 10-19
- $I_2 + I_2^1$, Age 20-29
- $I_2 + I_2^1$, Age 30-39
- $I_2 + I_2^1$, Age 40-49
- $I_2 + I_2^1$, Age 50-59
- $I_2 + I_2^1$, Age 60-69
- $I_2 + I_2^1$, Age 70+



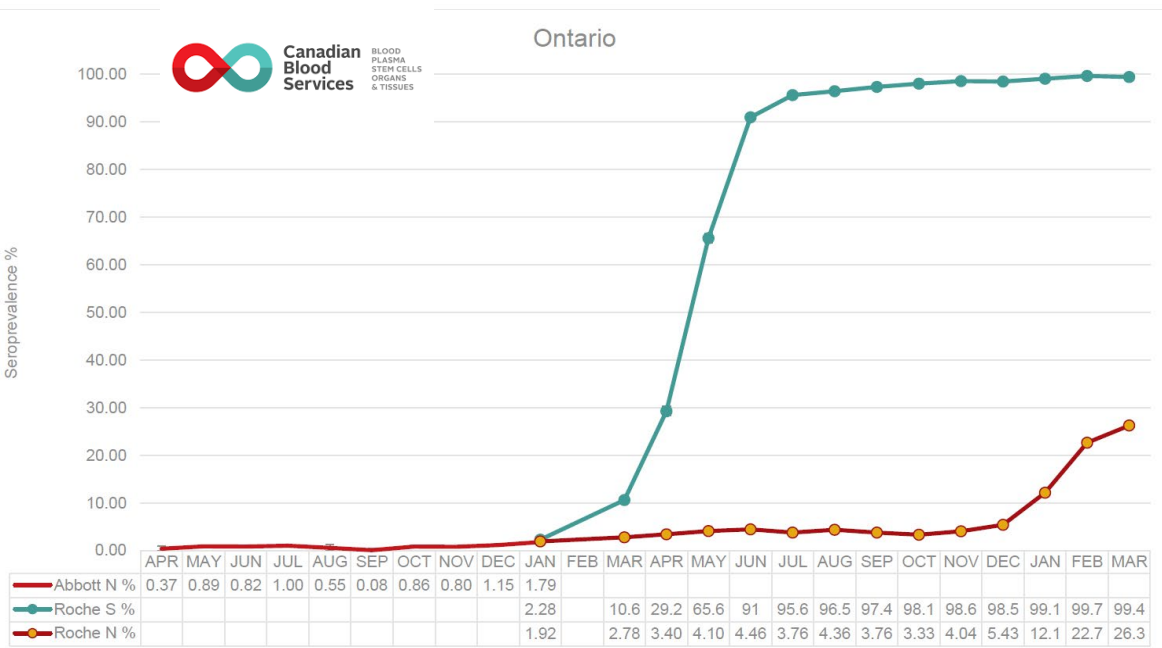
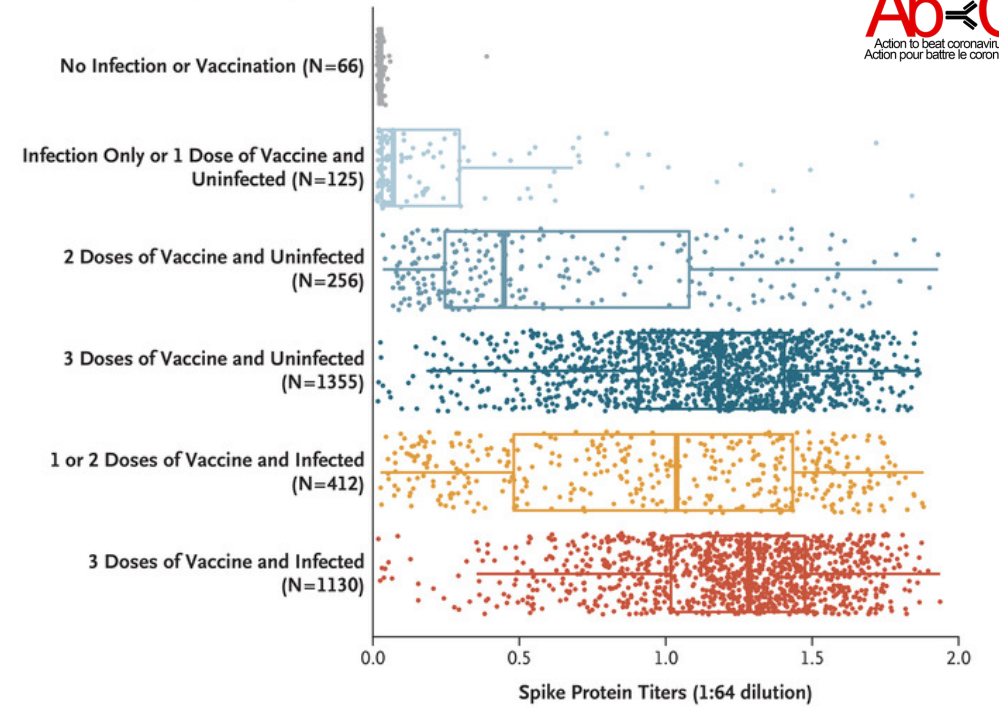
- $I_4 + I_4^1$, Age 70+
- $I_3 + I_3^1$, Age 70+
- $I_2 + I_2^1$, Age 70+
- $I_4 + I_4^1$, Age 60-69
- $I_3 + I_3^1$, Age 60-69
- $I_2 + I_2^1$, Age 60-69
- $I_4 + I_4^1$, Age 50-59
- $I_3 + I_3^1$, Age 50-59
- $I_2 + I_2^1$, Age 50-59
- $I_4 + I_4^1$, Age 40-49
- $I_3 + I_3^1$, Age 40-49
- $I_2 + I_2^1$, Age 40-49
- $I_4 + I_4^1$, Age 30-39
- $I_3 + I_3^1$, Age 30-39
- $I_2 + I_2^1$, Age 30-39
- $I_4 + I_4^1$, Age 20-29
- $I_3 + I_3^1$, Age 20-29
- $I_2 + I_2^1$, Age 20-29
- $I_4 + I_4^1$, Age 10-19
- $I_3 + I_3^1$, Age 10-19
- $I_2 + I_2^1$, Age 10-19
- $I_4 + I_4^1$, Age 0-9
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- $I_2 + I_2^1$, Age 0-9

Age

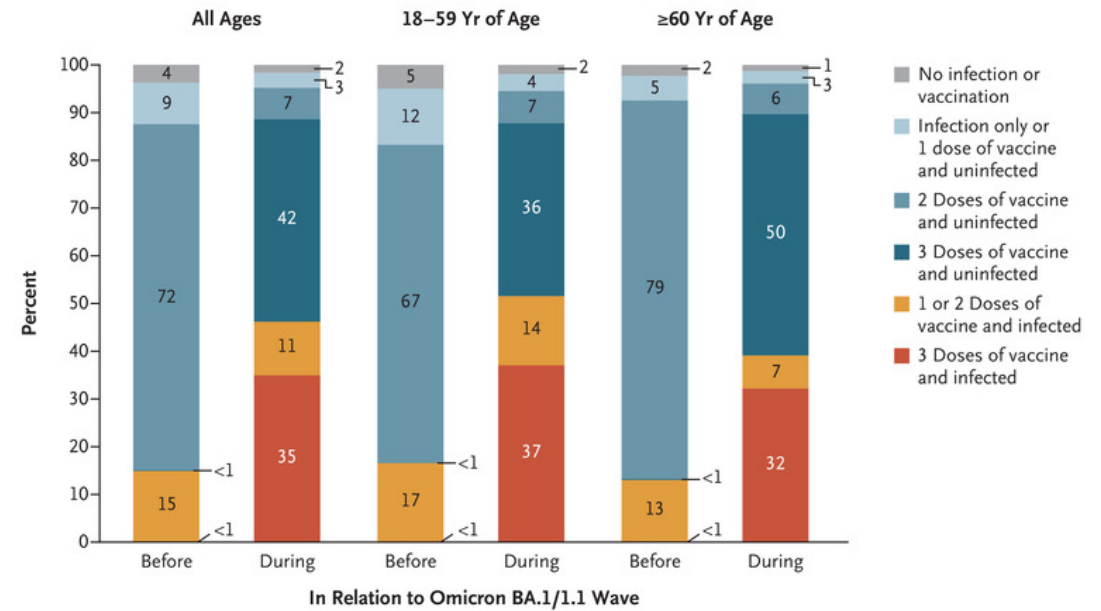




A Spike Protein Titers for IgG Antibody



B Cumulative Incidence of Infection before and during the Omicron BA.1/1.1 Wave



Reading

- Heffernan and Ciupe
- Wodarz and Nowak
- Perelson and Nelson

Homework Problem 1 – 10 points

- Consider the basic model of in-host infection with the virus loss term AND an eclipse stage split into 2 parts
- Draw a flow diagram
- Describe model variables and parameters
- Derive the basic reproduction number
- Find the model equilibria (fixed points)
- Show that the infected equilibrium can only exist if the basic reproduction number is greater than 1

$$\frac{dx}{dt} = \lambda - d_x x - \beta xv$$

$$\frac{de_1}{dt} = \beta xv - d_e e_1 - \alpha e_1, \quad \frac{de_2}{dt} = \alpha e_1 - d_e e_2 - \alpha e_2$$

$$\frac{dy}{dt} = \alpha e_2 - d_y y$$

$$\frac{dv}{dt} = ky - \beta xv - d_v v$$

Homework Problem 2 – 8 points

- Again, consider the basic model of in-host infection with the virus loss term AND an eclipse stage split into 2 parts (see Homework Problem 1)
- Rewrite the model to take into consideration infectious and non-infectious virus particles. Justify the structure that you chose.
- Add an equation for antibodies. Add in the neutralization for virus particles by antibodies. Justify why you added antibodies in the structure that you chose.
- Add an equation for cytotoxic T-cells. Add in the killing of infected cells. Justify why you added CTL in the format that you chose.
- Add an equation for Interferon and add in terms that represent the effects of interferon into the model. Justify your choices.

Homework Problem 3 – 4 points

- Read
 - Heffernan, J. M., and M. J. Keeling. "An in-host model of acute infection: Measles as a case study." *Theoretical population biology* 73.1 (2008): 134-147.
 - Heffernan, J. M., and M. J. Keeling. "Implications of vaccination and waning immunity." *Proceedings of the Royal Society B: Biological Sciences* 276.1664 (2009): 2071-2080
- Comment on the utility of embedding in-host information into epidemiological models (1 page)