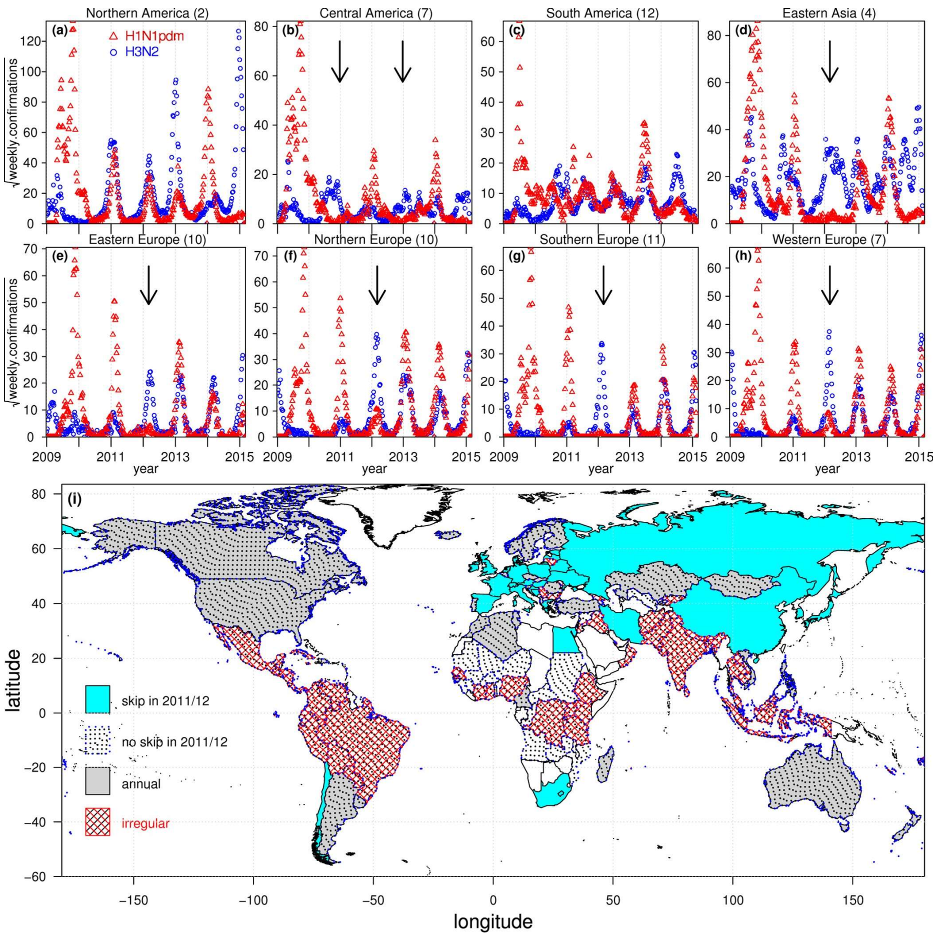
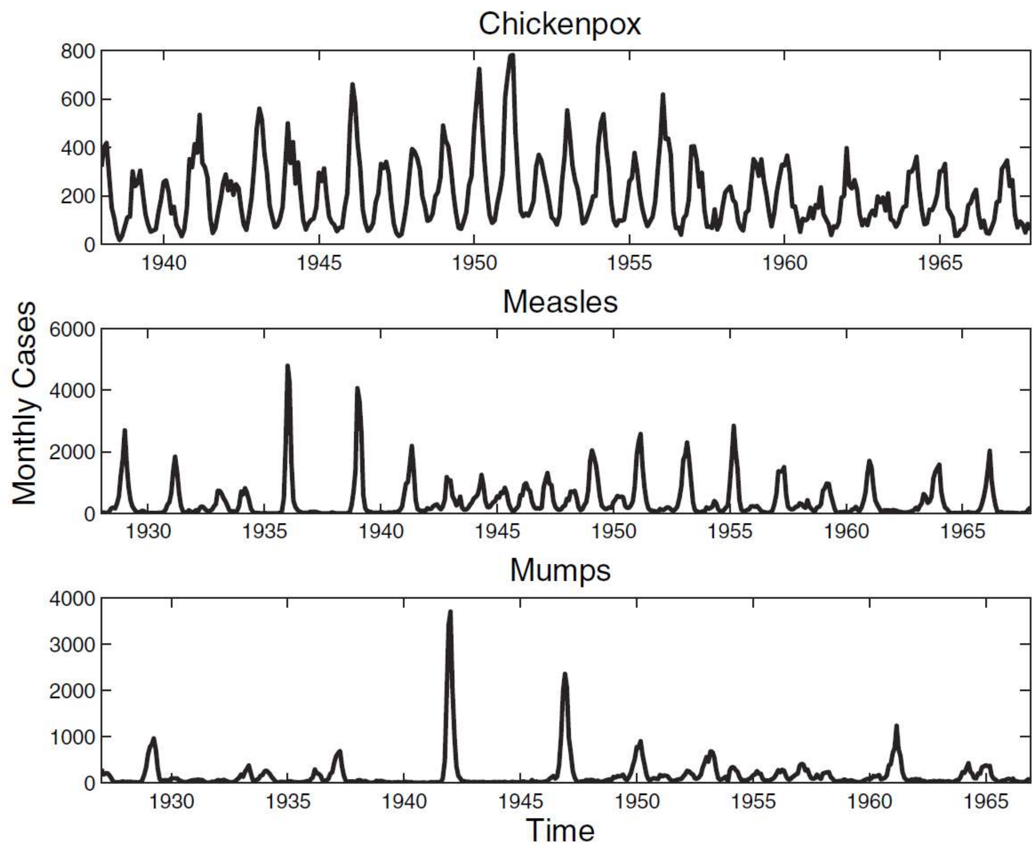
Seasonal Forcing

Periodicity of Infectious Diseases





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Incorporate Seasonality??

- Add in periodicity
  - Model during school and between schools years – age the children
  - Add a time varying transmission rate

SEIR

- 4 age groups

$SEIR$-type model is therefore:

$$\frac{dS_i}{dt} = \nu_i n_4 - \sum_j \beta_{ij} I_j S_i - \mu_i S_i,$$
$$\frac{dE_i}{dt} = \sum_j \beta_{ij} I_j S_i - \sigma E_i - \mu_i E_i,$$
$$\frac{dI_i}{dt} = \sigma E_i - \gamma I_i - \mu_i I_i,$$
$$\frac{dR_i}{dt} = \gamma I_i - \mu_i R_i,$$

and at the start of the school year, moving up an age group is controlled by:

$$Q_1 = Q_1 - Q_1/6$$
$$Q_2 = Q_2 + Q_1/6 - Q_2/4$$
$$Q_3 = Q_3 + Q_2/4 - Q_3/10$$
$$Q_4 = Q_4 + Q_3/10$$

where $Q \in \{S, E, I, R\}$.

$$\beta = \gamma \begin{pmatrix} 2.089 & 2.089 & 2.086 & 2.037 \\ 2.089 & 9.336 \pm 4.571 & 2.086 & 2.037 \\ 2.086 & 2.086 & 2.086 & 2.037 \\ 2.037 & 2.037 & 2.037 & 2.037 \end{pmatrix}.$$

- Baseline transmission β_0
- Period of forcing ω
- Amplitude of seasonality β_1
- $\beta(t) = \beta_0(1 + \beta_1 \cos(\omega t))$

$$\frac{dX}{dt} = \mu N - \beta(t) XYN,$$
$$\frac{dY}{dt} = \beta(t) XY/N - \gamma Y.$$

Seasonality

- In the absence of seasonal forcing, the SIR family of models exhibit a stable equilibrium
- Adding time-dependent transmission can generate
  - Annual epidemics
  - Multiennial outbreaks
  - Chaos

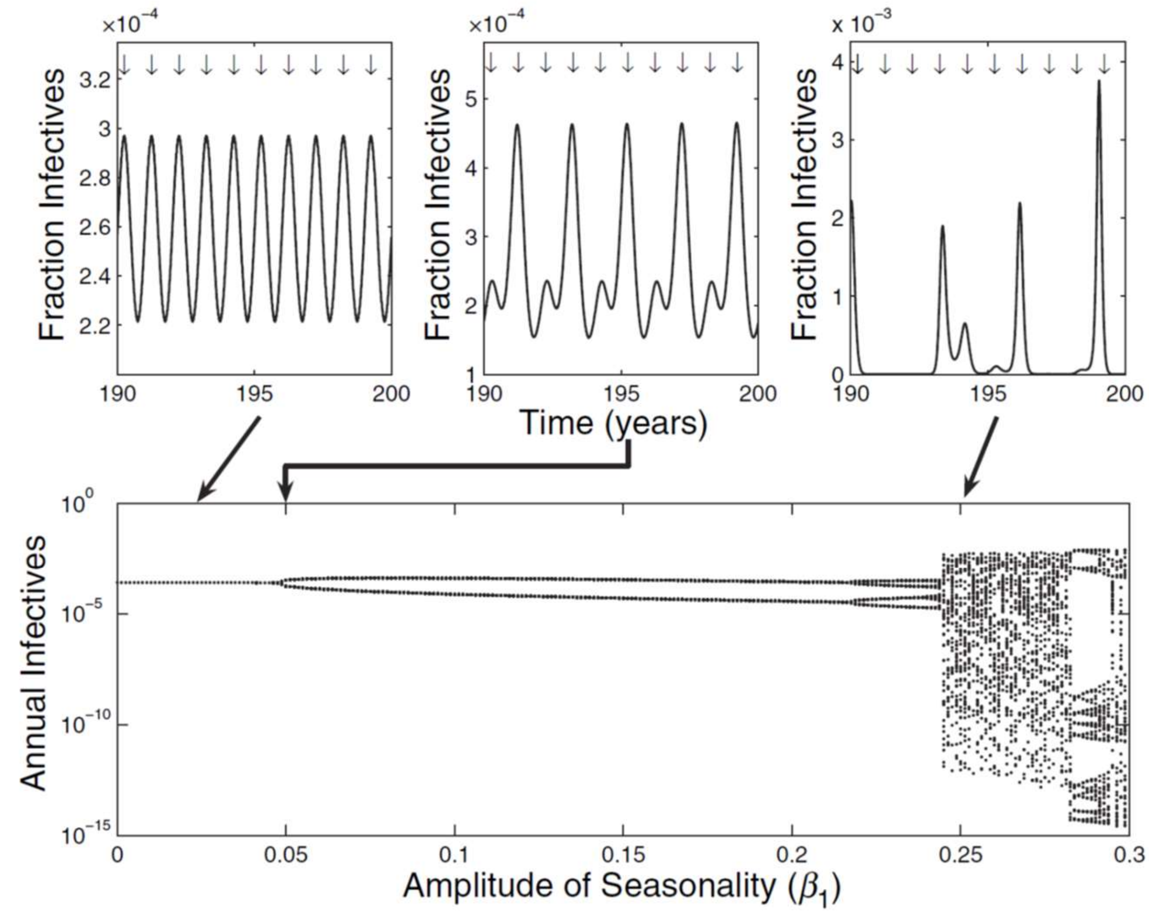


Figure 5.8. Constructing a bifurcation diagram. The top three panels depict time-series data for the $SEIR$ model with different levels of seasonality ($\beta_1 = 0.025, 0.05$, and 0.25, respectively). The arrows at the top of the panels indicate the points when the time series are sampled in order to construct the bifurcation diagram below. The parameters used to generate these panels were $\mu = 0.02$ per year, $\beta_0 = 1250$, $1/\sigma = 8$ days, and $1/\gamma = 5$ days. All simulations were started with $S(0) = 6 \times 10^{-2}$ and $E(0) = I(0) = 10^{-3}$.

SIRWS Model

- Birth and death - ξ
- Infection - β
- Recovery -
- Waning - κ

- Discuss
  - Seasonal forcing
  - Demographic change
  - Immune boosting

- Relative factor of
  osting – v>1
  ls with
  ate immunity
  ve heightened
  to boosting

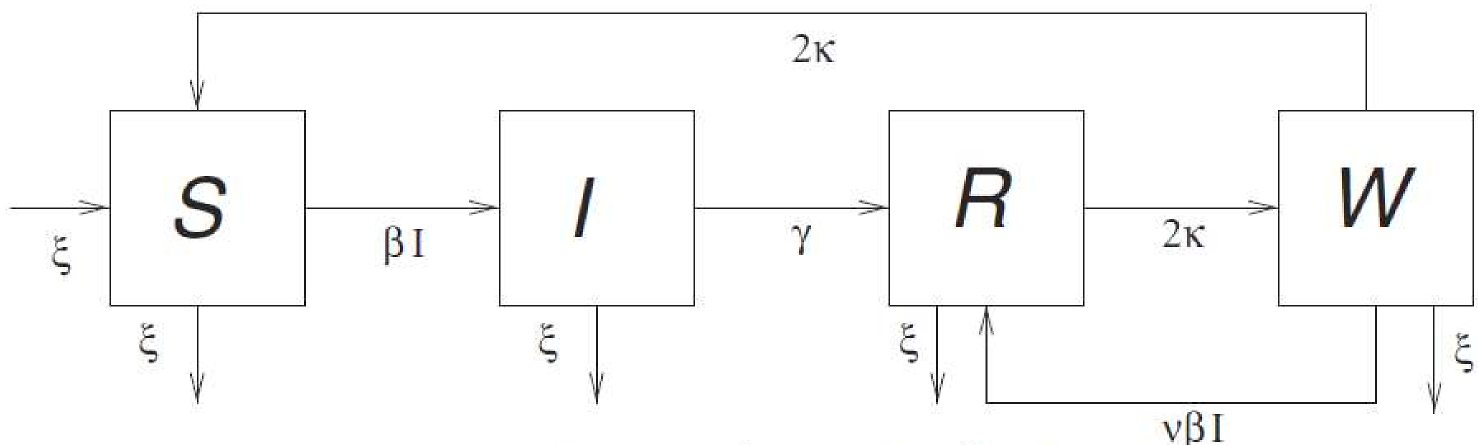


Figure 1 The SIRWS model.

$$\beta(t) = \beta_0(1 + \eta \cos(2\pi t))$$

Dafilis M, Frascoli F, et al. (2014) *Jour Theor Biol*

Immune-boosting and demography

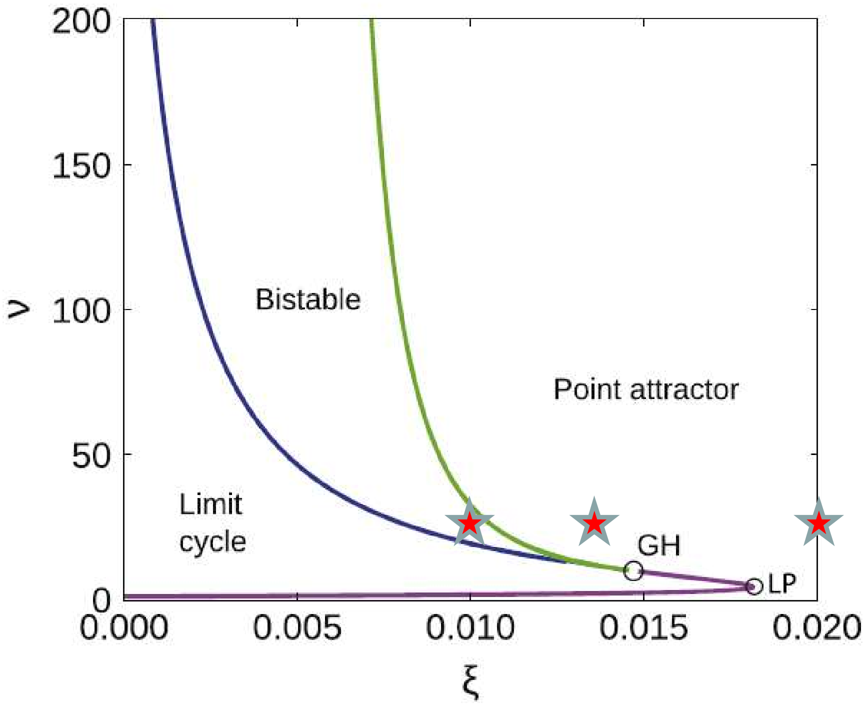


Fig. 2. Stability properties of the unforced SIRWS system. A two parameter bifurcation diagram in the (ξ, ν)–plane. Lines demarcate the different regions in parameter space. The Hopf bifurcation is subcritical for $\xi < \xi_{GH}$ and supercritical otherwise, allowing for bistable behaviour for $\xi < \xi_{GH}$. The value of ξ at the kink is given by $\xi_{LP} \approx 0.01814$, and at the GH by $\xi_{GP} \approx 0.01473$. [Reproduced from Dafilis et al., 2012].

Life expectancy ξ vs. immune boosting ν

No seasonal forcing here

★ Life expectancy of 50, 75, 100 years

Vary seasonal forcing η and immune boosting ν

$$\beta(t) = \beta_0(1 + \eta \cos(2\pi t))$$

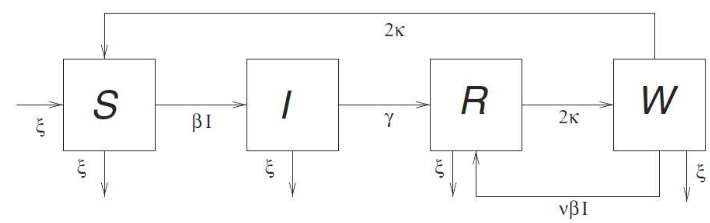
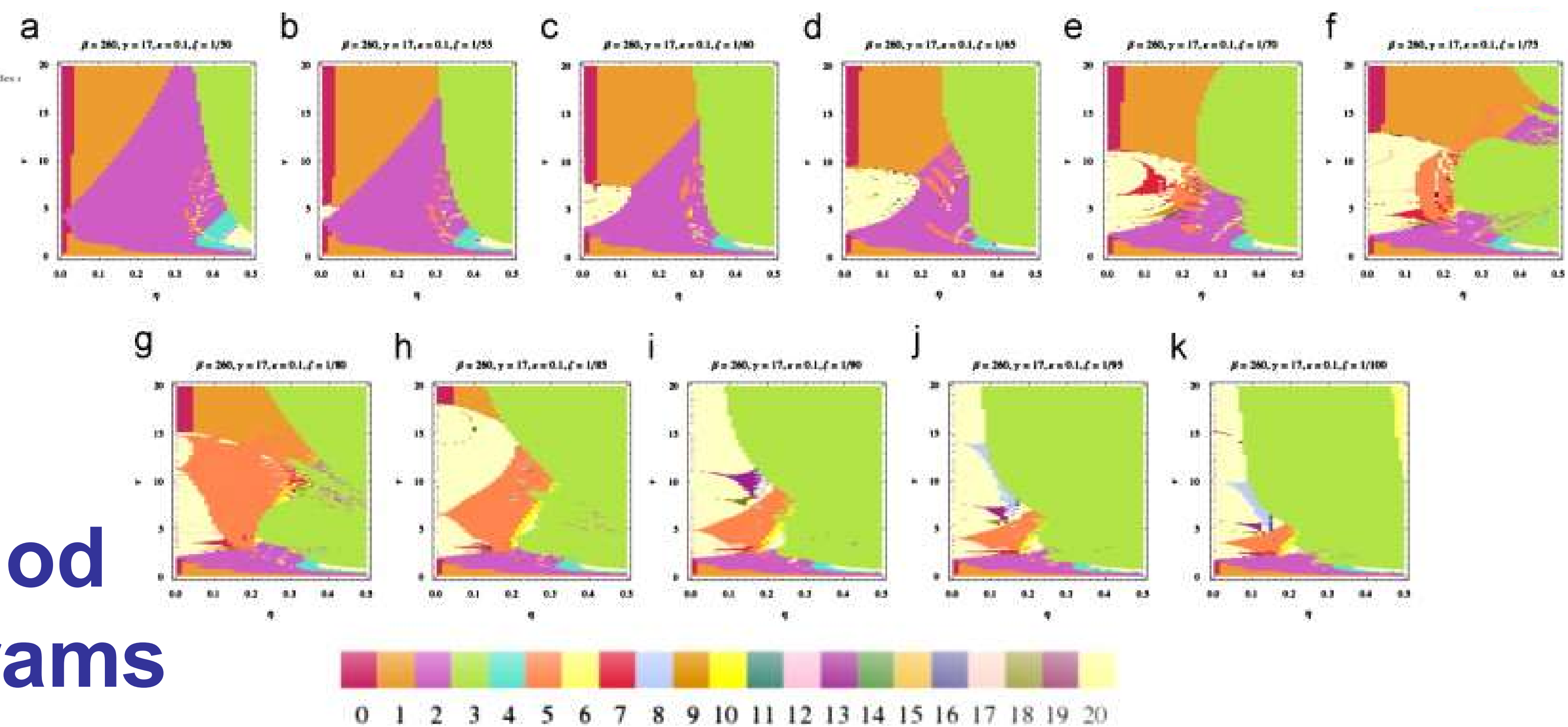


Figure 1 The SIRWS model.

Dafilis M, Frascoli F, et al. (2014) *Jour Theor Biol*



Period Diagrams

- Limit cycles and multistability in the unforced system give rise to complex and intricate behaviour as seasonal forcing is introduced
- Identified chaotic regions of parameter space
- Identified regions where saddle node lines and period doubling cascades of different orbital period overlap, suggesting that the observed behaviour of the SIRWS system is particularly responsive to small perturbations in its parameters

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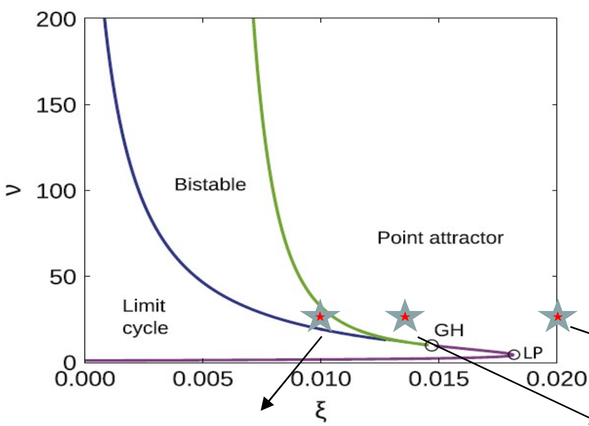
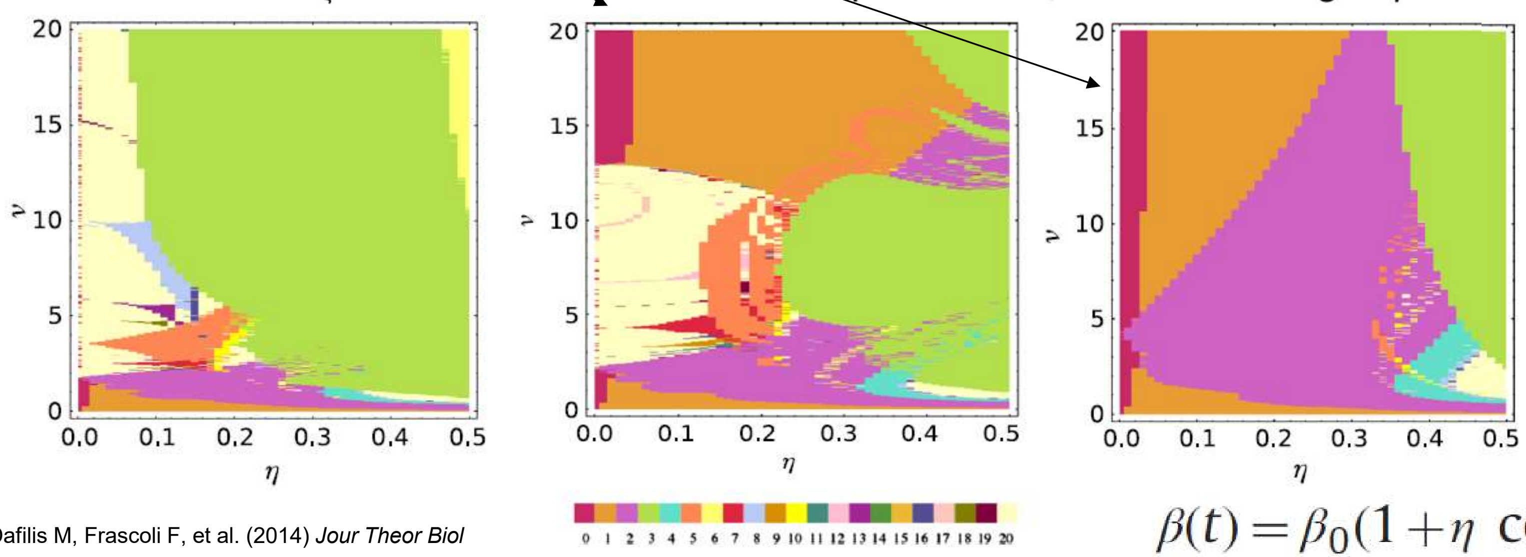
Immune-boosting and demography



- Gradual enlargement of period 20 regions for small η (from right to left)
- Potential for chaotic dynamics to appear under conditions of weak seasonal forcing as birth-rate decreases (life expectancy increases)
- Period 3 region encompasses more of the plane for moderate and larger η



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$$\beta(t) = \beta_0(1 + \eta\ \cos(2\pi t))$$

50 years

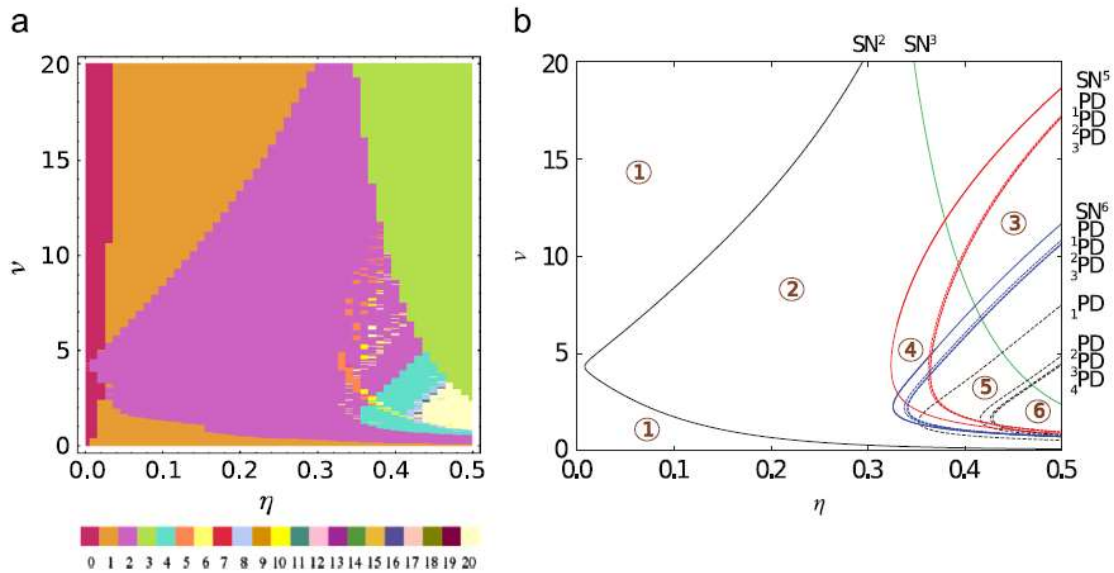


Fig. 4. Period and bifurcation diagrams for $\xi = 1/50$. (a) The period diagram in the (η, ν)–plane. Different colours represent different stable periodic orbits. For very weak seasonal forcing ($\eta \approx 0$) the system remains characterised by the unforced system's point attractor dynamics (see Fig. 2). It exhibits small amplitude in-phase annual fluctuations about the unforced system's equilibrium trajectory (red) for all values of the boosting (ν). For larger values of the forcing the trajectories may be entrained into stable limit cycles. We observe a number of different periods as discussed in detail in the main text. (b) The corresponding two parameter diagram for bifurcation points of forced orbits in the (η, ν)–plane. Saddle node lines (SN^m, solid) denote the boundary for areas in which period m orbits may be sustained. Period doubling lines ($_n\mathrm{PD}^m$, dashed) indicate where nth-generation orbits of the period-m cascade are present. The regions labelled ① through ⑥ are used to guide the discussion in the main text. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

50 years

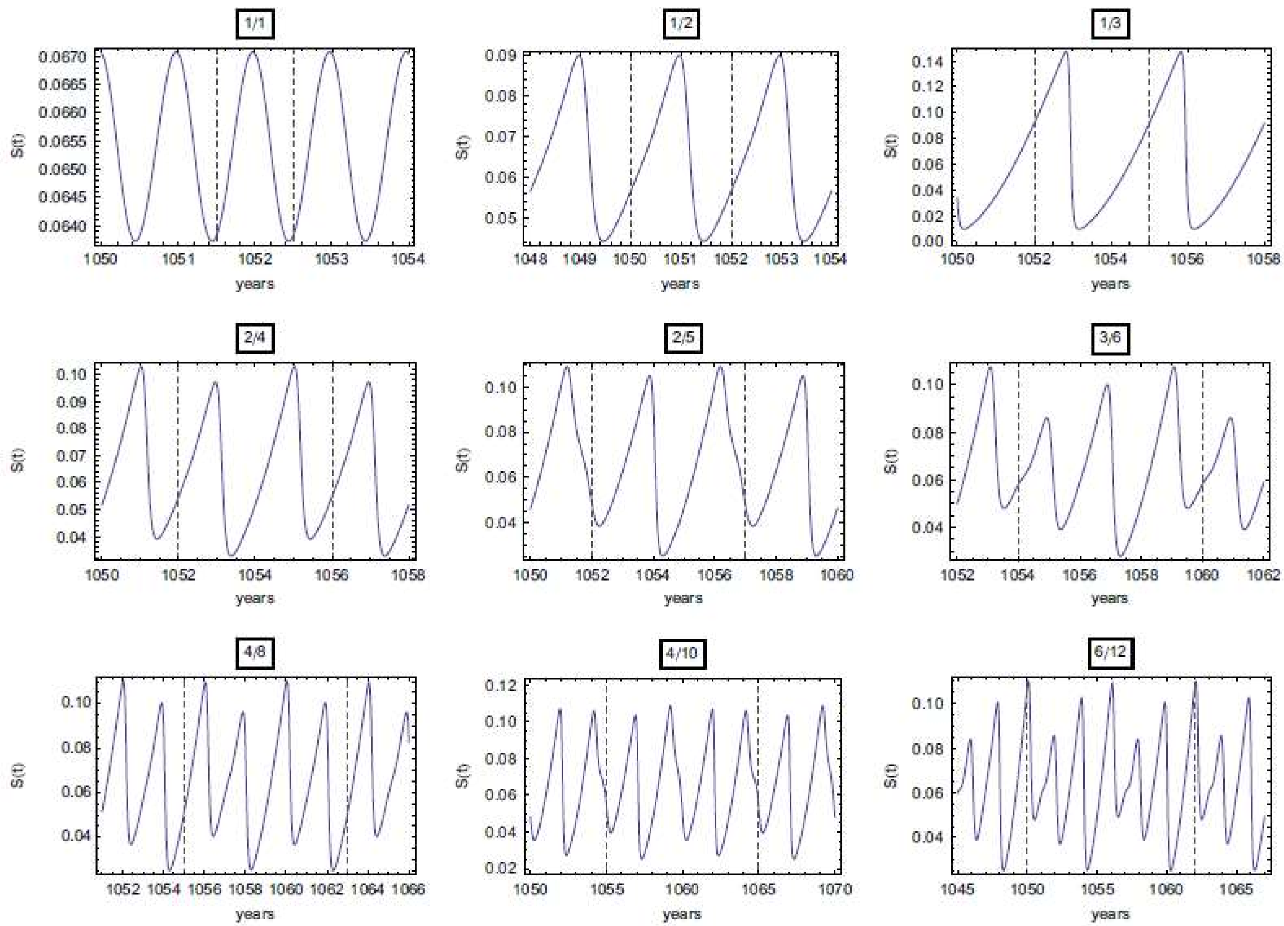
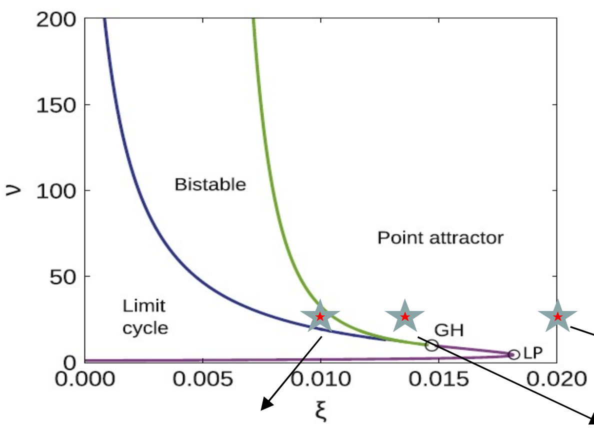


Fig. 5. Time series for $\xi = 1/50$. Examples of all the cycles occurring for the case $\xi = 1/50$ that have been found in the period diagram (Fig. 4). For a n/m cycle, vertical lines delimit the period m of each orbit, whereas the number of maxima over the period are indicated by n.

Periods 3, 5, and 20+



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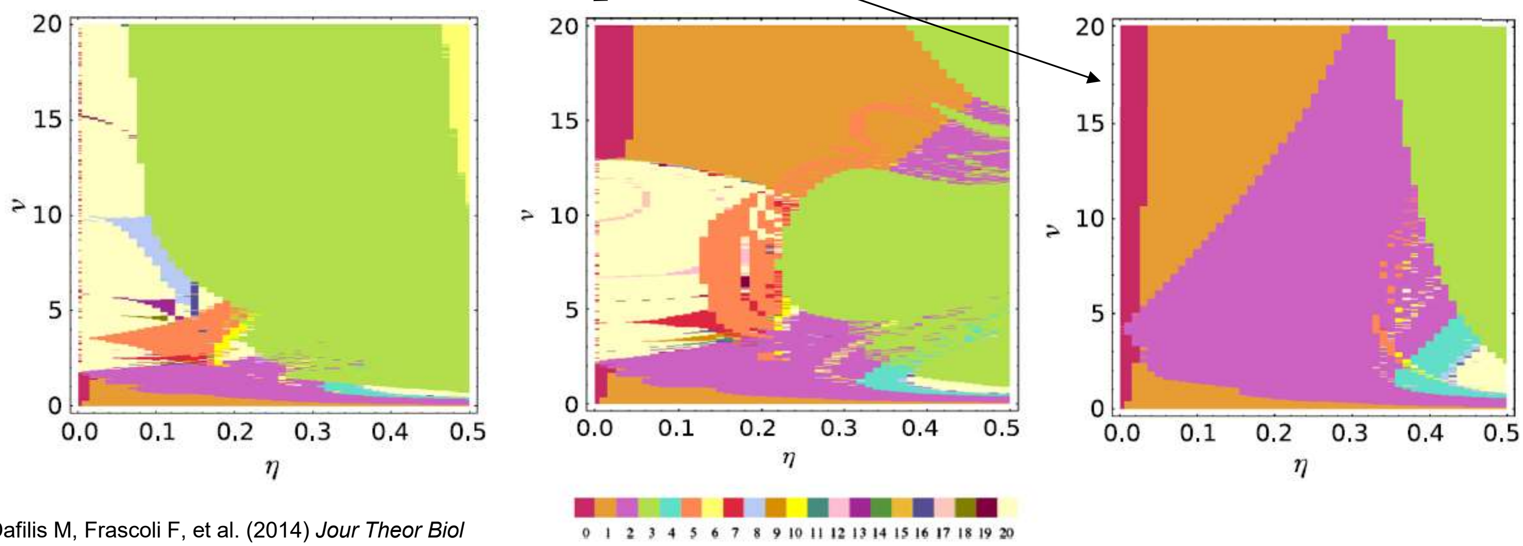

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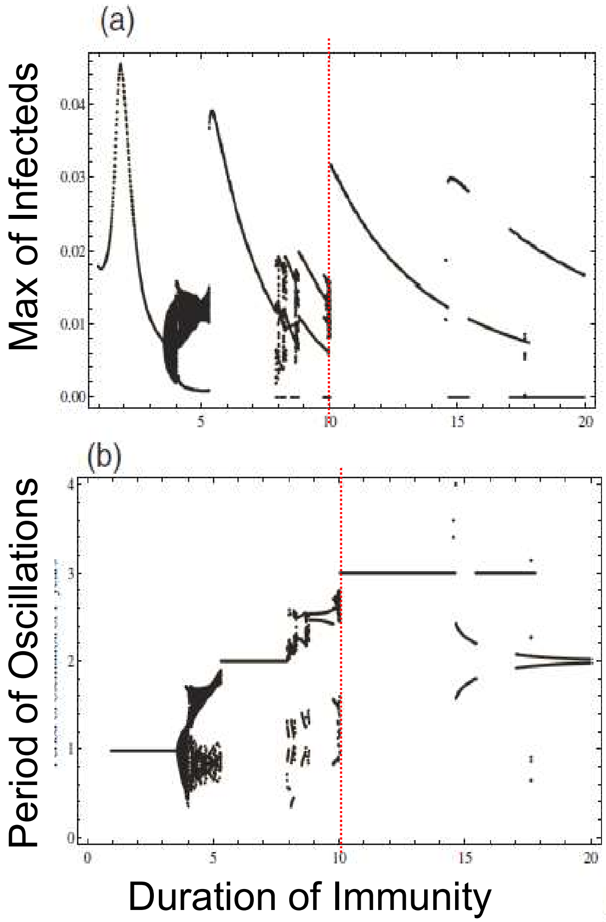
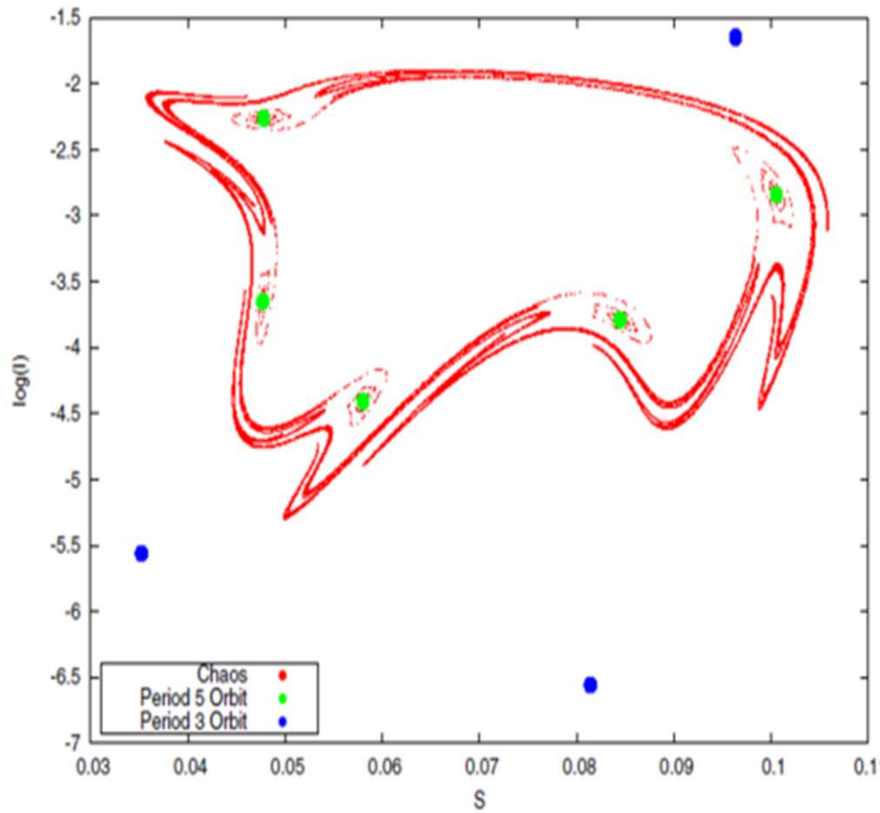
Crises and Chaos

- Basins of attraction, vary duration of immunity





Dafilis M, Frascoli F, et al. (2014) *Theor Biol Med Model* 11:43.

$\beta_0 = 260$ years^{-1}, $\gamma = 17$ years^{-1}, $\kappa = 1/10$ years^{-1}, $\xi = 1/100$ years^{-1} and $\nu = 5$

SIRWS Model

- Birth and death - ξ
- Infection - β
- Recovery - γ
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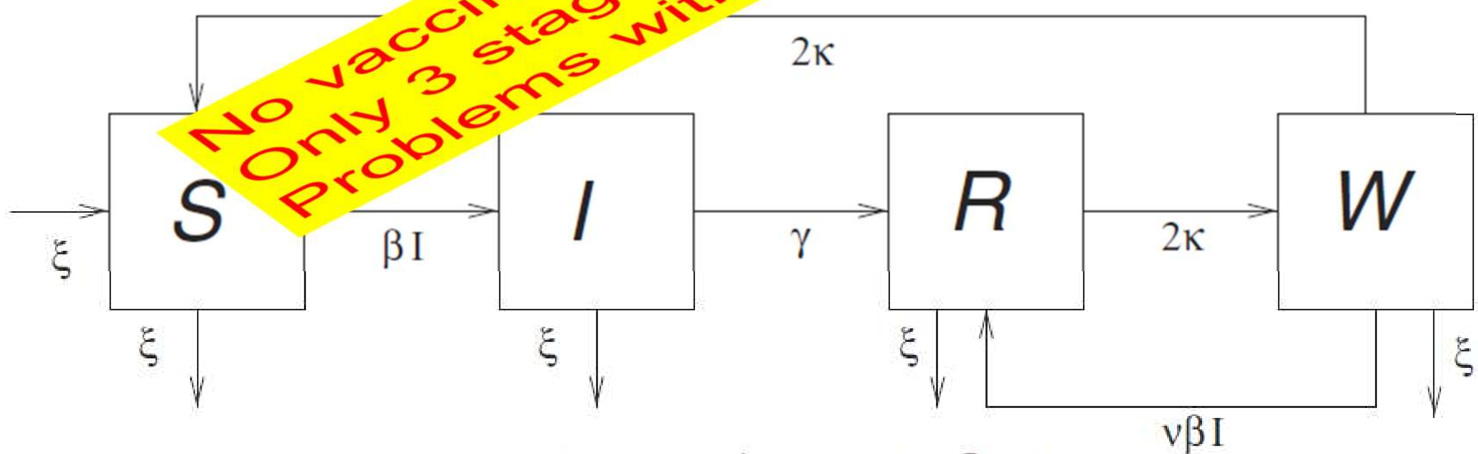
- Discuss
  - Seasonal forcing
  - Demograp
  - Imm

- Relative factor of
  ν>1
  ate immunity
  ve heightened
  / to boosting

No vaccine
Only 3 stages of immunity
Problems with assumption on ν



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Lavine et al., Earn et al, Dafilis M, JTB (2014)